## UV PHYSICS ACADEMY
### STATISTICAL MECHANICS

**SM(1)**

Classical Thermodynamics :

First law $\delta Q = dU + \delta W$, Entropy $dS = \frac{\delta Q}{T}$, $dS = 0$ for reversible process.

Mathematical form of second law $\delta Q = TdS$

**For solids and liquids :**

$Q = mL$ ; $L_V = 536$ cal/gm (water) $L_F = 80$ cal/gm (ice)

**For an ideal gas:**

$$ds = ms\ln\left(\frac{T_f}{T_i}\right) = 2.303\ ms\log_{10}\frac{T_f}{T_i}$$

$$ds = n\left[c_v\ln\left(\frac{P_f}{P_i}\right) + c_p\ln\left(\frac{V_f}{V_i}\right)\right]$$

$$ds = n\left[c_p\ln\left(\frac{T_f}{T_i}\right) - R\ln\left(\frac{P_f}{P_i}\right)\right]$$

$$ds = n\left[c_v\ln\left(\frac{T_f}{T_i}\right) + R\ln\left(\frac{V_f}{V_i}\right)\right]$$

$\delta Q$ = heat given to a system (It is not a state function)
du = change in energy (It is a state function)
$\delta w$ = work done on a system (It is not a state function)
m = mass of substance
L = latent heat
v = vapour
F = fusion
C = heat capacity
i = initial state, f = final state
n = number of moles
P = pressure

Adiabatic process: $PV^\gamma = $ const; $P^{1-\gamma}T^\gamma = $ const; $TV^{\gamma-1} = $ const

Polytropic process: $PV^n = $ const
Isothermal: n=1; Isobaric : n=0
Isochoric: $n = \infty$; Adiabatic: $n = \gamma$

Specific heat (C): $C = C_V + \frac{R}{1-n}$

## UV PHYSICS ACADEMY

**SM(3)**

Thermodynamic potentials :

$du = TdS - PdV + \mu dN$

$H = U + PV$ ; $dH = TdS + VdP + \mu dN$

$F = U - TS$ ; $dF = -SdT - PdV + \mu dN$

$G = F + PV = H - TS$ ; $dG = -SdT + VdP + \mu dN$

S = entropy
$\mu$ = chemical potential
N = number of particles
H = enthalpy
P = pressure
F = Helmholtz free ener
G = Gibbs free energy

Maxwell's relations :

Maxwell 1 : $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V\left(= \frac{\partial^2 U}{\partial S\partial V}\right)$ ; Maxwell 2 : $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P\left(= \frac{\partial^2 H}{\partial P\partial S}\right)$

Maxwell 3 : $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T\left(= \frac{\partial^2 F}{\partial T\partial V}\right)$ ; Maxwell 4 : $\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T\left(= \frac{\partial^2 G}{\partial P\partial T}\right)$

Heat capacities :

$C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$ ; $C_P = \left(\frac{\partial Q}{\partial T}\right)_P = \left(\frac{\partial H}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_P$ ; Ideal gas $C_P - C_V = R$

Vanderwaal gas (real gas) $C_P - C_V = -T\left[\left(\frac{\partial P}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_P^2\right] = R\left[1 + \frac{2a}{RVT}\right]$

Joule - kelvin coefficient ($\mu$): $\mu = \left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$

## UV PHYSICS ACADEMY

**SM(5)**

Kinetic theory :

Classical equipartition theorem $E = \frac{1}{2}kT$

(average energy per atom per degree of freedom)

Monoatomic gas : $U = \frac{3}{2}NkT = \frac{N}{2}mC_{rms}^2$; $C_V = \frac{3}{2}Nk$, $C_P = \frac{5}{2}Nk$

Ideal gas : $C_V = \frac{1}{2}fNk = \frac{1}{2}fnR$ ; $C_P = Nk\left(1 + \frac{f}{2}\right)$

$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$

$C_p$ = most probable speed
$C_{rms}$ = root mean square speed
$\gamma$ = ratio of heat capacities

Transport properties :

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$ ; $\eta = \frac{1}{3}\rho\bar{C}\lambda$; $K = \frac{1}{3}\rho\bar{C}\lambda C_V = \eta\rho C_V$ ; $D = \frac{1}{3}\bar{C}\lambda = \frac{\eta}{\rho}$

$C_{rms} = \left(\frac{3kT}{m}\right)^{1/2} = \left(\frac{3RT}{M}\right)^{1/2}$ ; $\bar{C} = \left(\frac{8kT}{\pi m}\right)^{1/2} = \left(\frac{8RT}{\pi M}\right)^{1/2}$ ;

$C_P = \left(\frac{2kT}{m}\right)^{1/2} = \left(\frac{2RT}{M}\right)^{1/2}$ $P = \frac{1}{3}mn\bar{C}^2$

k = Boltzmann constant
$C_P$, $C_V$ = heat capacities at constant pressure, volume
T = absolute temperature
P = pressure
m = particle mass
n = number density = N/V
N = total no. of particles
$C_{rms}^2$ = mean squared particle velocity
f = no. of degrees of freedom
n = no. of moles
N = no. of molecules
R = molar gas constant
$\lambda$ = mean free path
$\eta$ = coefficient of viscocity
K = thermal conductivity
D = coefficient of self diffusion
d = molecular diameter
$\rho$ = density of gas molecules

## UV PHYSICS ACADEMY

**SM(7)**

Connection between TD & SM $S = K\ln\Omega$

Types of ensemble :

| Ensemble | Type | Macrostates |
|---|---|---|
| Microcanonical | Isolated | (N, E, V) |
| Canonical | Closed | (N, T, V) |
| Grand canonical | Open | ($\mu$, T, V) |

S = entropy
K = boltzmann const.
$\Omega$ = no. of acessible microstates
N = number of systems
$\mu$ = chemical potential
$\rho$ = density in phase space
$E_s$ = energy of system in state 's'

Volume of unit cell in phase space $V = h^{3N}$; dimensions = (energy × time)

Liouville's theorem $\frac{d\rho}{dt} = \{\rho, H\} + \frac{\partial\rho}{\partial t} = 0$

Note : This equation governs the motion of incompressible fluid in phase space.

Canonical ensemble : $P_s = \frac{e^{-\beta E_s}}{\sum_s e^{-\beta E_s}} = \frac{e^{-\beta E_s}}{Z}$ ; Partition function : $Z = \sum_s e^{-\beta E_s}$ ;

Gibbs entropy : $S = -K\sum_s P_s\ln P_s$ ; $P_s = -\frac{1}{\beta}\frac{\partial(\ln Z)}{\partial E_s}$

Mean energy (internal energy) : $U = F + TS = -\frac{\partial(\ln Z)}{\partial\beta}$

$\beta = \frac{1}{KT}$
$P_s$ = probability of findig the system in state 's'
Z = partition function

Helmholtz free energy $F = A = -KT\ln Z$; $S = -\left(\frac{\partial A}{\partial T}\right)_{V,N}$ ; $P = -\left(\frac{\partial A}{\partial V}\right)_{N,T}$ ; $\mu = \left(\frac{\partial A}{\partial N}\right)_{V,T}$

## UV PHYSICS ACADEMY

**SM(9)**

Quantum mechanically $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

or N - systems $Z_N = \left[2\sinh\left(\frac{\beta\hbar\omega}{2}\right)\right]^{-N}$

$= NkT\ln\left[2\sinh\left(\frac{\beta\hbar\omega}{2}\right)\right]$ $U = \frac{N\hbar\omega}{2}\coth\left(\frac{\beta\hbar\omega}{2}\right)$

$_V = Nk(\beta\hbar\omega)^2\,\mathrm{cosech}^2\left(\frac{\beta\hbar\omega}{2}\right)$ $P = 0$

$= Nk\left(\frac{\beta\hbar\omega}{e^{\beta\hbar\omega} - 1} - \ln(1 - e^{-\beta\hbar\omega})\right)$

$E_n$ = energy of harmonic oscillator
n = principle quantum number
N = number of systems
P = pressure
k = Boltzmann constant
Z = partition function
$\beta$ = 1/kT
F = Helmholtz function (energy)
S = entropy
T = temperature
U = internal energy (mean energy)
$C_v$ = heat capacity at constant volume

in 1/2 particle in a magnetic field : $Z_N = [2\cosh(\mu\beta H)]^N$

$= -NkT\ln[2\cosh(\mu\beta H)]$ $S = Nk(\ln[2\cosh(\mu\beta H) - \mu\beta H\tanh(\mu\beta H)])$

$= Nk(\mu\beta H)^2\,\mathrm{sech}^2(\mu\beta H)$ ; $U = -N\mu H\tanh(\mu\beta H)$

rie's law : $\chi = \frac{C}{T}$; $C = \frac{N\mu^2}{3k}$ $M = N\mu\tanh(\mu\beta H)$

## UV PHYSICS ACADEMY

**SM(11)**

Canonical partition function : $Z_{molecule} = Z_{trans} \cdot Z_{rot} \cdot Z_{vib} \cdot Z_{ele}$

Monoatomic partition function : $Z = \frac{V^N}{h^{3N}}(2\pi mkT)^{3N/2}$

**Diatomic molecules :** $Z_{trans} = Z_1 = \frac{V}{h^3}[2\pi(m_1 + m_2)kT]^{3/2}$

$$Z_{rot} = \sum_{j=0}^{\infty}(2J+1)e^{-\beta\left(\frac{\hbar^2}{2I}\right)J(J+1)} = \sum_{j=0}^{\infty}(2J+1)e^{-\frac{\theta_r}{T}J(J+1)};\ \theta_r = \frac{\hbar^2}{2KI}$$

$E_{rot}$ = rotational energy
I = moment of inertia of molecule
J = rotational quantum number
J = 0, 1, 2,3...
(2J + 1) = degeneracy of $J^{th}$ state
$m_1$, $m_2$ = masses of two atoms
$I = \left(\frac{m_1m_2}{m_1 + m_2}\right)r_0^2$

at high temperature : $Z_{rot} = \frac{2IkT}{\hbar^2}$ ; generally $Z_{rot} = \frac{2IkT}{\sigma\hbar^2}$ ; $E_{rot} = \frac{\hbar^2}{2I}J(J+1)$

$\sigma = $ { 1 hetero molecule; 2 diatomic (homo nuclear); 3 molecule with three fold systems }

$$Z_{ortho} = \sum_{odd J}(2J+1)e^{-J(J+1)\frac{\theta_r}{T}}$$

$Z_{total} = 3Z_{ortho} + Z_{para}$ $C_V = \frac{3}{4}C_{V(ortho)} + \frac{1}{4}C_{V(para)}$ $Z_{para} = \sum_{even J}(2J+1)e^{-J(J+1)\frac{\theta_r}{T}}$

**Ortho and para hydrogen:**

Ortho : Both spins are aligned parallel and spin wave function is symmetric
Para : Both spins are aligned antiparallel and spin wave function is antisymmetric
Note : Rotational wave function is symmetric for even values of J and anti symmetric for odd values of J.

## UV PHYSICS ACADEMY

**SM(4)**

$\gamma = \frac{C_P}{C_V} = \frac{K_T}{K_S} = \frac{E_S}{E_T}$

Phase transitions:

Gibbs phase rule P + F = C + 2

Clausius – Clapayron equation $\frac{dP}{dT} = \frac{S_2 - S_1}{V_2 - V_1} = \frac{L}{T(V_2 - V_1)}$

Co-existance curve $P(T) \propto \exp\left(-\frac{L}{RT}\right)$

R = molar gas constant
$K_T$ = isothermal compressibility
$K_S$ = adiabatic compressibility
$E_S$ = adiabatic bulk modulus
$E_T$ = Isothermal bulk modulus
P = no. of phases in equilibrium
F = no. of degrees of freedom
C = no. of components
P = pressure
T = temperature
L = Latent heat
1, 2 = initial, final states
$\beta_P$ = Isobaric expansivity

Thermodynamic coefficients :

$\beta_P = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$ ; $K_T = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ ; $K_S = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_S$ ; $E_T = \frac{1}{K_T} = -V\left(\frac{\partial P}{\partial V}\right)_T$ ; $E_S = \frac{1}{K_S} = -V\left(\frac{\partial P}{\partial V}\right)_S$

TdS equations :

Ist TdS equation $TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$; $S = S(T,V)$

2nd Tds equation $TdS = C_P dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$; $S = S(T,P)$

Specific heats: $C_p - C_V = R$
$\gamma = C_P / C_V$   $C_V = \frac{R}{\gamma - 1}$   $C_P = \frac{\gamma R}{\gamma - 1}$
$\gamma = 1 + \frac{2}{f}$   $C_P = R(1 + f/2)$
$C_V = fR/2$

## UV PHYSICS ACADEMY

**SM(2)**

| Parameter | Isobaric (p=const.) | Isochoric (v = const.) | Isothermal (T = const.) | Adiabatic (Q = const. & s = const.) |
|---|---|---|---|---|
| dw | pdv = nRdT | zero | $nRT \ln\left(\frac{V_2}{V_1}\right)$<br>$P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$<br>$nRT \ln\left(\frac{P_1}{P_2}\right)$ | $\frac{nR}{\gamma - 1}(T_1 - T_2)$<br>$nC_v(T_1 - T_2)$<br>$\frac{nC_P}{\gamma}(T_1 - T_2)$ , $\frac{(P_1V_1 - P_2V_2)}{\gamma - 1}$ |
| dQ | $mc_PdT$ / $nc_PdT$ | $mC_vdT$ | dw | zero |
| dU | dQ – PdV,<br>dQ – nR dT | $mC_vdT$ | zero | – dw |
| Equation<br>of state | $\frac{V_1}{T_1} = \frac{V_2}{T_2}$<br>$\frac{V}{T}$ = constant | $\frac{P_1}{T_1} = \frac{P_2}{T_2}$<br>$\frac{P}{T}$ = const. | $P_1V_1 = P_2V_2$<br>PV = const. | $PV^\gamma$ = const.<br>$TV^{\gamma-1}$ = const.<br>$P^{1-\gamma}T^\gamma$ = const. |

## UV PHYSICS ACADEMY

**SM(8)**

Ideal gas in C.E. : (No interaction among particles)   $H(q_i, p_i) = \sum_{i=1}^{3N} \frac{P_i^2}{2m}$

$Z_{dist} = \frac{1}{h^{3N}} \int e^{-\beta H} d^{3N}q\, d^{3N}p$   $Z_{indist} = \frac{Z_{dist}}{N!} = \frac{1}{N!h^{3N}} \int e^{-\beta H} d^{3N}q\, d^{3N}p$

$Z_{ideal} = \frac{V^N}{N!}\left(\frac{2\pi mKT}{h^2}\right)^{3/2} = \frac{V^N}{N!}\left(\frac{1}{\lambda}\right)^{3N}$

$F = NKT\left[\ln\left(\frac{N}{V}\left(\frac{h^2}{2\pi mKT}\right)^{3/2}\right) - 1\right]$

$P = \frac{NKT}{V}$, $U = \frac{3}{2}NKT$; $C_V = \frac{3}{2}NK$

$S = NK\left[\ln\left(\frac{V}{N}\left(\frac{2\pi mKT}{h^2}\right)^{3/2}\right) + \frac{5}{2}\right]$

$\mu = KT \ln\left(\frac{N}{V}\left(\frac{h^2}{2\pi mKT}\right)^{3/2}\right)$

N! = Gibbs factor
$\lambda = \left(\frac{h^2}{2\pi mKT}\right)^{1/2}$
mean free path

change in entropy of ideal gases : $\Delta S = NK \ln\left[\left(\frac{T_2}{T_1}\right)^{3/2}\left(\frac{V_2}{V_1}\right)\right]$

Note : Gibbs factor represents indistinguishable character.

**Linear Harmonic oscillator :**   $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2q^2$

**Classically**   $Z_1 = \frac{1}{(\beta\hbar\omega)}$; $Z_N = \left(\frac{1}{\beta\hbar\omega}\right)^N$   $F = NkT \ln[\beta\hbar\omega]$

$S = Nk\left[1 + \ln\left(\frac{kT}{\hbar\omega}\right)\right]$   $U = NkT$   $C_V = Nk$   $P = 0$

## UV PHYSICS ACADEMY

**SM(6)**

Maxwell Boltzmann distribution : $\frac{N(c)}{N} = \left(\frac{m}{2\pi KT}\right)^{3/2} . e^{\frac{-mc^2}{2KT}} 4\pi C^2 dC$

Wien's law of distribution of energy : $E_\lambda = \frac{8\pi hC}{\lambda^5} e^{\frac{-hC}{\lambda KT}}$;  $\lambda < \lambda_m$

Rayleigh - Jean's law : $E_\lambda = \frac{8\pi KT}{\lambda^4}$;  $\lambda > \lambda_m$

Planck's law : $E_\lambda = \frac{8\pi hC}{\lambda^5\left(e^{\frac{h\upsilon}{KT}} - 1\right)}$ (valid at all wavelengths)

N(C) = no. of molecules having velocity C.
N = total number of molecules
C = velocity of a molecule
$E_\lambda$ = monochromatic energy
u = energy density

diffuse radiation $P = \frac{1}{3}u$ ;  perfect reflector $P = \frac{2}{3}u$ (diffuse radiation); P = u (plane waves)

Avg. energy of Planck's oscillator $\bar{E} = \frac{h\upsilon}{\left(e^{\frac{h\upsilon}{KT}} - 1\right)}$ ; Wavelength at maximum energy : $\lambda_m = \frac{hc}{5KT}$

Average energy of classical oscillator : $\bar{E} = KT$

Number of modes of vibration (no. of degrees of freedom) $N_\lambda d\lambda = \frac{8\pi}{\lambda^4} d\lambda$ or $N_\upsilon d\upsilon = \frac{8\pi\upsilon^2}{C^3} d\upsilon$

## UV PHYSICS ACADEMY

**SM(12)**

**Vibrational partition function:**

Vibrational motion : $E_v = \left(v + \frac{1}{2}\right)\hbar\omega$ ; $Z_{vib} = \frac{e^{-\frac{\beta\hbar\omega}{2}}}{1 - e^{-\beta\hbar\omega}} = e^{-\frac{\theta_v}{2}T}\left[1 - e^{-\frac{\theta_v}{2}T}\right]^{-1}$

$<E> = -N\frac{\partial(\ln Z)}{\partial\beta} = NKT^2\frac{\partial(\ln Z)}{\partial T}$;  $<E> = NK\left[\frac{\theta_V}{2} + \frac{\theta_V}{e^{\frac{\theta_v}{T}} - 1}\right]$

$E_V$ = vibrational energy of a molecule
v = vibrational quantum number
$\omega$ = angular frequency of a molecule
$\theta_V = \frac{\hbar\omega}{K}$ vibrational characterstic temp.

$C_V = NK\left(\frac{\theta_V}{T}\right)^2 . \frac{e^{\frac{\theta_v}{T}}}{\left(e^{\frac{\theta_v}{T}} - 1\right)^2}$ ; at high temperature $\frac{\theta_V}{T} << 1$; $<E> = NKT + \frac{N\hbar\omega}{2}$ ; $C_V = NK$

Electronic partition function : $Z_{el} = g_0 + g_1e^{-\beta\varepsilon_1} + g_2e^{-\beta\varepsilon_2} + ...$

with sufficient accuracy $Z_{ele} = g_0 + g_1e^{-\beta\varepsilon_1}$ ; $<E> = N\varepsilon_1\left(\frac{g_0}{g_1}e^{\beta\varepsilon_1} + 1\right)^{-1}$

$\varepsilon_1$, $\varepsilon_2$ = energies required for excitation of electron to the various levels.
g = respective degeneracies

The total temperature dependent energy of the gas : $E = \left(\frac{3}{2}NKT\right)_{trans} + (NKT)_{rot} + (NKT)_{vib}$

$E = \frac{7}{2}NKT$ ; the total specific heat $C = \frac{7}{2}NK$

Note : There is no significant contribution from the electronic energy to the specific heat except at high temperatures.

## UV PHYSICS ACADEMY

**SM(10)**

A magnetic dipole placed in a uniform magnetic field : $Z = \frac{4\pi}{\beta\mu H}\sinh(\mu\beta H)$

For N non-interacting dipoles : $Z_N = \left[\left(\frac{4\pi}{\beta\mu H}\right)\sinh(\beta\mu H)\right]^N$

$F = -NkT \ln\left[\frac{4\pi}{\beta\mu H}\sinh(\mu\beta H)\right]$   $M = N\mu\left[\coth(\mu\beta H) - \frac{1}{\mu\beta H}\right]$

$<\mu> = \frac{M}{N} = \mu L(\mu\beta H)$   Langevin function $L(x) = \coth x - \frac{1}{x}$

$\mu$ = intrinsic magnetic dipole moment
H = magnetic field
M = total magnetic moment
N = no. of spin 1/2 particle
$\chi$ = susceptibility
k = Boltzmann constant
$<\mu>$ = Mean magnetic moment

**Equipartition theorem :**

Hamiltonian depends quadratically on a momentum or a coordinate contributes a mean energy 1/2 kT.

3D H.O. $<E> = 6 \times \frac{1}{2}kT = 3kT$ ; L.H.O $<E> = kT$ ; Total energy $E = N<E>$

**Thermodynamic probabilities**

F.D : $W = \prod_i \frac{g_i!}{n_i!(g_i - n_i)!}$; $g_i \geq n_i$   $S_{MB} = k\sum_i \ln\left(\frac{Ng_i}{n_i}\right)^{n_i}$   $S_{FD} = k\sum_i\left[n_i \ln\left(\frac{g_i}{n_i} - 1\right) - g_i \ln\left(1 - \frac{n_i}{g_i}\right)\right]$

B.E : $W = \prod_i \frac{(n_i + g_i - 1)!}{n_i!(g_i - 1)!}$ ; M.B : $W = N!\prod_i \frac{(g_i)^{n_i}}{n_i!}$   $S_{BE} = k\sum_i\left[n_i \ln\left(\frac{g_i}{n_i} + 1\right) + g_i \ln\left(1 + \frac{n_i}{g_i}\right)\right]$

## UV PHYSICS ACADEMY

**SM(13)**

Quantum statistics : $\langle n_s \rangle = \frac{1}{e^{\beta(\epsilon_s-\mu)}+a}$; $a = \begin{cases} +1 & \text{F.D} \\ -1 & \text{B.E} \\ 0 & \text{M.B} \end{cases}$

Quantum statistics gives M.B statistics when $e^{-\beta\mu} >> 1$

Limiting case : $\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\frac{V}{N} >> 1$

Non-degenerate gas: $\left(\frac{2\pi mKT}{h^2}\right)^{3/2}\frac{V}{N} >> 1$

Degenerate gas: $\left(\frac{h^2}{2\pi mKT}\right)^{3/2}\frac{N}{V} >> 1$

Density matrix $\rho_{mn}(t) = \frac{1}{N}\sum_{k=1}^{N}\{a_m^k(t)a_n^{*k}(t)\}$

An electron in a magnetic field $\rho = \frac{1}{e^{\beta H\mu_B}-e^{-\beta H\mu_B}}\begin{pmatrix} e^{\beta H\mu_B} & 0 \\ 0 & e^{-\beta H\mu_B}\end{pmatrix}$

$\langle\sigma_Z\rangle = Tr(\rho\sigma_Z) = Tanh(\beta H\mu_B)$ $\quad \rho_{Canonical} = e^{-\beta\epsilon}/Trace(e^{-\beta\epsilon})$

$\langle M_Z\rangle = \mu_B\langle\sigma_Z\rangle = \mu_B \tanh(\beta H\mu_B)$

$\chi = \frac{\partial\langle M_Z\rangle}{\partial H} = \beta\mu_B^2\,\mathrm{sech}^2(\beta H\mu_B)$ $\quad \langle G\rangle = \frac{\text{Trace of }(\rho G)}{\text{Trace of }(\rho)}$

$g_i$ = degeneracy of $i^{th}$ state;
$n_i$ = number of cells in $i^{th}$ state;
$N_r$ = no. of particles in $r^{th}$ state;
$E_s$ = energy of $s^{th}$ state
$\alpha = -\mu/kT$;
q = potential ;
$\rho_{mn}$ = density matrix, $z = e^{-\alpha N}$
<G> =expectation value of G;
$\langle M_Z\rangle$ = avg. value of total magnetic moment.

**Grand canonical ensemble:**

$P_{r,s} = \frac{e^{-\alpha N_r - \beta E_s}}{\sum e^{-\alpha N_r-\beta E_s}}$ $\quad + \to$ F.D

$Z_{GC} = \prod_k\left(1 \pm ze^{-\beta\epsilon_k}\right)^{\pm}$ $\quad - \to$ B.E

$q = \frac{1}{a}\sum_K \ln(1+aze^{-\beta\epsilon_k})$

## UV PHYSICS ACADEMY
### SOLID STATE PHYSICS

**SSP(1)**

Crystalline structure :

volume of primitive cell : $v = (\bar{a}\times\bar{b})\cdot\bar{c}$ Reciprocal primitive base vectors : $a^* = 2\pi\frac{\bar{b}\times\bar{c}}{(\bar{a}\times\bar{b})\cdot\bar{c}}$

$b^* = 2\pi\frac{\bar{c}\times\bar{a}}{(\bar{a}\times\bar{b})\cdot\bar{c}}$ $\quad c^* = 2\pi\frac{\bar{a}\times\bar{b}}{(\bar{a}\times\bar{b})\cdot\bar{c}}$ $\quad \bar{a}\cdot\bar{a}^* = \bar{b}\cdot\bar{b}^* = \bar{c}\cdot\bar{c}^* = 2\pi$ $\quad \bar{a}\cdot\bar{b}^* = \bar{a}\cdot\bar{c}^* = \bar{b}\cdot\bar{c}^* = 0$

Lattice vector $\bar{R}_{uvw} = u\bar{a} + v\bar{b} + w\bar{c}$ ;

Reciprocal lattice vector $\bar{G}_{hk\ell} = h\bar{a}^* + k\bar{b}^* + \ell\bar{c}^*$

$\exp i(\bar{G}_{hk\ell}\cdot\bar{R}_{uvw}) = 1$; Weiss zone equation : $hu + kv + \ell w = 0$

Inter planar spacing (general) $d_{hk\ell} = \frac{2\pi}{G_{hk\ell}}$

Inter planar spacing (orthogonal basis) $\frac{1}{d_{hk\ell}^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{\ell^2}{c^2}$

Note : Miller Indices are defined so that $G_{hk\ell}$ is the shortest reciprocal lattice vector normal to the $(hk\ell)$ planes.

Geometrical structure factor $F = \sum_{j=1}^{n} f_j e^{i\bar{G}\cdot\bar{r}_j} = \sum_{j=1}^{n} f_j e^{2\pi i(u_j h + v_j k + w_j l)}$

$\bar{a},\bar{b},\bar{c}$ = primitive base vectors
$\bar{a}^*,\bar{b}^*,\bar{c}^*$ = reciprocal primitive base vectors
$\bar{R}_{uvw}$ = lattice vector
u, v, w = integers $i^2 = -1$
$(hkl)$ miller indices of planes
$d_{hk\ell}$ = distance between $(h\,k\,\ell)$ planes

$(h^2+k^2+l^2)$ values for allowed reflections

SC: 1:2:3:4:5;6:8:9:10
FCC: 3:4:8:11:12:16:19:20
BCC: 1:2:3:4:5:6:7:8:9
DC : 3:8:11:16:19

FCC : $F = f[1 + e^{i\pi(h+k)} + e^{i\pi(k+l)} + e^{i\pi(l+h)}]$
BCC : $F = f[1 + e^{i\pi(h+k+l)}]$
SCC: $F(h,k,l) = f$

## UV PHYSICS ACADEMY

**SSP(3)**

The lattice parameters for the reciprocal lattices of SC, BCC and FCC are $\frac{2\pi}{a}$, $\frac{4\pi}{a}$ and $\frac{4\pi}{a}$ respectively

Primitive base vectors :

| simple cubic | body centred | face centred |
|---|---|---|
| $\bar{a}_1 = a\hat{x}$ | $\bar{a}_1 = \frac{a}{2}(\hat{y}+\hat{z}-\hat{x})$ | $\bar{a}_1 = \frac{a}{2}(\hat{y}+\hat{z})$ |
| $\bar{a}_2 = a\hat{y}$ | $\bar{a}_2 = \frac{a}{2}(\hat{z}+\hat{x}-\hat{y})$ | $\bar{a}_2 = \frac{a}{2}(\hat{z}+\hat{x})$ |
| $\bar{a}_3 = a\hat{z}$ | $\bar{a}_3 = \frac{a}{2}(\hat{x}+\hat{y}-\hat{z})$ | $\bar{a}_3 = \frac{a}{2}(\hat{x}+\hat{y})$ |

reciprocal lattice vectors for bcc
$a_1^* = \frac{2\pi}{a}(i+j)$
$a_2^* = \frac{2\pi}{a}(j+k)$
$a_3^* = \frac{2\pi}{a}(k+i)$

reciprocal lattice vectors for fcc
$a_1^* = \frac{2\pi}{a}(i+j-k)$
$a_2^* = \frac{2\pi}{a}(-i+j+k)$
$a_3^* = \frac{2\pi}{a}(i-j+k)$

$\bar{k}$ = propogation vector
$\bar{G}$ = reciprocal lattice vector = $h\bar{a}^* + k\bar{b}^* + \ell\bar{c}^*$
$\bar{S}_0$ = unit vector in the direction of incident beam
$\bar{S}$ = unit vector in the direction of diffracted beam
$\alpha_0,\alpha$ are the angles b/w crystal plane and incident beam, diffracted beam respectively.
e, f, g = integers
n = order of diffraction

**Crystal diffraction:**

Bragg's law in vector form $2\bar{K}\cdot\bar{G} + \bar{G}^2 = 0$

Laue equations $a(\cos\alpha - \cos\alpha_0) = \bar{a}\cdot(\bar{S}-\bar{S}_0) = e\lambda$

$b(\cos\beta - \cos\beta_0) = \bar{b}\cdot(\bar{S}-\bar{S}_0) = f\lambda$ $\quad c(\cos\gamma - \cos\gamma_0) = \bar{c}\cdot(\bar{S}-\bar{S}_0) = g\lambda$

Atomic factor (scattering factor) $f(\bar{G}) = \int_{vol} e^{-i\bar{G}\cdot\bar{r}}\rho(r)\,d^3r$ ; $\bar{r} = u\bar{a} + v\bar{b} + w\bar{c}$

Bragg's law (condition for diffraction) $2d\sin\theta = n\lambda$ $\quad$ scattered intensity $I \propto N^2|F|^2$

## UV PHYSICS ACADEMY

**SSP(5)**

**Extinction rules for cubic crystal:**

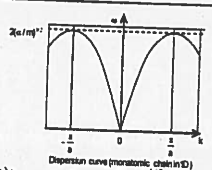
| Crystal | Allowed reflections |
|---|---|
| SC | for all value of $(h^2+k^2+\ell^2)$ |
| BCC | for even values of $(h+k+\ell)$ |
| FCC | when h, k and $\ell$ are all odd (or) all even |
| DC | when h,k and $\ell$ are all odd (or) all even, $(h+k+\ell)$ should be divisible by four |

$\omega$ = phonon angular frequency; $\alpha$ = spring constant
m = atomic mass; $v_p$ = phase velocity; $v_g$ = group velocity
k $(=2\pi/\lambda)$= wave number; a = atomic separation; DC = diamond cubic

**Lattice dynamics :**

Monatomic linear chain:

Dispersion relation $\omega = \left(\frac{4\alpha}{m}\right)^{1/2}\sin\left(\frac{ka}{2}\right)$



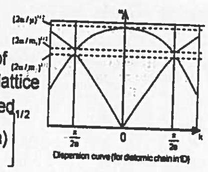
Dispersion curve (monatomic chain in 1D)

for short wavelength limit : $v_p = \frac{\omega}{k} = a\left(\frac{\alpha}{m}\right)^{1/2}\left(\frac{\sin(ka/2)}{ka/2}\right)$; $v_g = \frac{d\omega}{dk} = a\left(\frac{\alpha}{m}\right)^{1/2}\cos(ka/2)$

for long wavelength limit : $\omega = ka\left(\frac{\alpha}{m}\right)^{1/2}$, $v_p = a\left(\frac{\alpha}{m}\right)^{1/2} = V_g$

Note : 1. Normal modes (Dof) are equivalent to the number of atoms or no. of unit cells. 2. Under the low wavelength limit lattice acts a lowpass filter only frequencies $0 < \omega \le \omega_m$ are transmitted

Diatomic linear chain $(m_1 > m_2)$ : $\omega^2 = \frac{\alpha}{\mu} \pm \alpha\left[\frac{1}{\mu^2} - \frac{4}{m_1m_2}\sin^2(ka)\right]^{1/2}$



Dispersion curve (for diatomic chain in 1D)

## UV PHYSICS ACADEMY

**SSP(7)**

Debye theory :

Mean energy per phonon mode $\langle E\rangle = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}$

Debye frequency $\omega_D = v_s\left(\frac{6N\pi^2}{V}\right)^{1/3}$ ; where $\frac{3}{v_s^3} = \frac{1}{v_\ell^3} + \frac{2}{v_t^3}$

Debye $T^3$ law $C_v \approx \frac{12\pi^4}{5}Nk\frac{T^3}{\theta_D^3}$ $(T << \theta_D)$ (at Low Temp)

Dulong and petit's law $C_v \approx 3Nk$ $(T >> \theta_D)$ (at high temp)

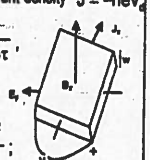
$g(\omega)d\omega = \frac{3V\omega^2}{2\pi^2 v_s^3}d\omega$ for $0 < \omega < \omega_D$, $g(\omega)d\omega = 0$ otherwise $\quad \theta_D = \frac{\hbar\omega_D}{k}$

Electrons in solids :

**Free electron transport properties :** current density $\bar{J} = -ne\bar{v}_d$

$\bar{v}_d = -\frac{e\tau}{m_e}\bar{E}$, $\quad \sigma_0 = \frac{ne^2\tau}{m_e}$, $\quad \sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}$

$\lambda = \frac{1}{3}\frac{C_v}{V}\bar{C}^2\tau = \frac{\pi^2 nk\tau T}{3m_e}$ $(T << T_F)$.



Wiedemann - Franz law : $\frac{\lambda}{\sigma T} = L = \frac{\pi^2k^2}{3e^2}$;

Hall voltage (rectangular strip) $V_H = R_H\frac{B_zI_x}{W}$; $R_H = \frac{1}{ne} = \frac{E_y}{J_xB_z}$

$v_s$=effectivesoundvelocity
N = number of atoms in the crystal
V = crystal volume
$v_\ell$ = longitudenal phase velocity
$C_v$ = heat capacity at constant volume
$g(\omega)\,d\omega$ = phonon density of states
n = freeelectron no. density
$\sigma_0$ = DC electrical conductivity
$\sigma(\omega)$ = AC electrical conductivity
$E_y$=Hall electric field
$T_F$=Fermi temperature
$\tau$ = mean time between successive collisions
L = Lorentz number
$J_x$ = applied current density
$I_x$ = applied current
w = strip thickness
$B_z$ = magnetic field

## UV PHYSICS ACADEMY

**SSP(9)**

### Fermigas :

$\epsilon = \frac{\hbar^2k^2}{2m} \Rightarrow |k| = \sqrt{\frac{2m|\epsilon|}{\hbar^2}}$ ; $E = 2\int_0^{\epsilon_f}\rho(\epsilon)\epsilon\,d\epsilon$ ; $N = 2\int_0^{\epsilon_f}\rho(\epsilon)d\epsilon$ ; $\rho(k)dk = \rho(\epsilon)d\epsilon$

| Parameter | 1 - dimension | 2 - dimension | 3 - dimension |
|---|---|---|---|
| Density $\rho(k)$ | $\frac{1}{(2\pi/L)}$ | $\frac{2\pi\lvert k\rvert}{(2\pi/L)^2}$ | $\frac{4\pi\lvert k\rvert^2}{(2\pi/L)^3}$ |
| Total energy(E) | $\frac{L}{3\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\epsilon_f^{3/2}$ | $\frac{L^2}{2\pi}\frac{m}{\hbar^2}\epsilon_f^2$ | $\frac{L^3}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon_f^{5/2}$ |
| Total no. of Fermions(N) | $\frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\epsilon_f^{1/2}$ | $\frac{L^2}{\pi}\frac{m}{\hbar^2}\epsilon_f$ | $\frac{L^3}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon_f^{3/2}$ |
| Number density | N/L | N/L² | N/L³ |
| Avg. energy $\left(\bar{E} = \frac{E}{N}\right)$ | $\frac{1}{3}\epsilon_f$ | $\frac{1}{2}\epsilon_f$ | $\frac{3}{5}\epsilon_f$ |
| Fermi energy | $(n\pi)^2\left(\frac{\hbar^2}{2m}\right)$ | $(2n\pi)\left(\frac{\hbar^2}{2m}\right)$ | $(3n\pi^2)^{2/3}\left(\frac{\hbar^2}{2m}\right)$ |

## UV PHYSICS ACADEMY

SSP(2)

Cubic Lattices :

| Lattice | simple cubic (primitive) | Body centred I (BCC) | Face centred F(FCC) | Hexagonal closed pack | Diamond |
|---|---|---|---|---|---|
| Lattice parameter | a | a | a | a | a |
| Volume of unit cell | $a^3$ | $a^3$ | $a^3$ | $a^3 3\sqrt{2}$ | $a^3$ |
| No. of lattice points per unit cell | 1 | 2 | 4 | 6 | 8 |
| Atomic radius | $\frac{a}{2}$ | $\frac{a\sqrt{3}}{4}$ | $\frac{a}{2\sqrt{2}}$ | $\frac{a}{2}$ | $\frac{a\sqrt{3}}{8}$ |
| 1st nearest neighbour distance | a | $\frac{a\sqrt{3}}{2}$ | $\frac{a}{\sqrt{2}}$ | a | $\frac{a\sqrt{3}}{4}$ |
| 2nd nearest neighbour distance | $a\sqrt{2}$ | a | a | $a\sqrt{\frac{8}{3}}$ | $\frac{3}{4}a$ |
| Co-ordination number | 6 | 8 | 12 | 12 | 4 |
| Packing fraction | 52% | 68% | 74% | 74% | 34% |
| Reciprocal lattice | SCC | FCC | BCC | | |

Note : for a close -packed spheres : The maximum possible packing fraction is $\frac{\sqrt{2}\cdot\pi}{6}$

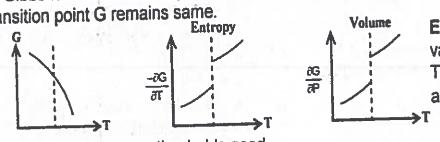
## UV PHYSICS ACADEMY

SM(14)

### Phase Transitions

**First Order :**

Here Gibbs function w.r.t. temp. and pressure change discontinuously at transition point. At transition point G remains same.

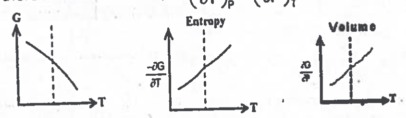



Eg: Transition of water into vapour at const temperature Transition of ice into water at 0°C and at 1 atmp.

Clasusius Clapeyron equation holds good.

**Second Order:**

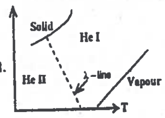
Here there is no discontinuity of $\left(\frac{\partial G}{\partial T}\right)_P$ & $\left(\frac{\partial G}{\partial P}\right)_T$ .Second order derivative change discontinuously




Eg: Transition of ferromagnetic material to paramagnetic material at curie temperature.

**λ - transition :**

Transition of liquid helium I to liquid helium II at 4.2 K is called λ - transition.




The l point line indicates that continuous phase transition has occured.

## UV PHYSICS ACADEMY

SSP(6)

Acoustic branch : $\omega_-^2 = \frac{\alpha}{\mu} - \alpha\left[\frac{1}{\mu^2} - \frac{4}{m_1 m_2}\sin^2(ka)\right]^{1/2}$

$(\omega_-)_{min} = ka\left[\frac{2\alpha}{m_1+m_2}\right]^{1/2}$ if $k \to 0$; $(\omega_-)_{max} = \left[\frac{2\alpha}{m_1}\right]^{1/2}$ if $k = \pi/2a$

Optical branch : $\omega_+^2 = \frac{\alpha}{\mu} + \alpha\left[\frac{1}{\mu^2} - \frac{4}{m_1 m_2}\sin^2(ka)\right]^{1/2}$

$(\omega_+)_{min} = \left[\frac{2\alpha}{m_2}\right]^{1/2}$ if $k = \pi/2a$ ; $(\omega_+)_{max} = \left[\frac{2\alpha}{\mu}\right]^{1/2}$ if $k = 0$

$m_1, m_2$ = atomic masses ($m_1 > m_2$)

$\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass

$\alpha_1, \alpha_2$ are alternating spring constant

a = atomic separation

Note : 1. The linear diatomic lattice behaves as a band pass mechanical filter.

2. The frequency band between the top of acoustic branch and bottom of the optical branch is called "forbidden band"

3. If 'n' atoms are in a primitive cell :-Total no. of branches = 3n out of which Acoustic branches = 3; Optical branches = 3n – 3

Identical masses, alternating spring constants :

$$\omega^2 = \frac{\alpha_1+\alpha_2}{m} \pm \frac{1}{m}\left(\alpha_1^2+\alpha_2^2+2\alpha_1\alpha_2 \cos ka\right)^{1/2};$$

$$\omega^2 = \begin{cases} 0, \frac{2(\alpha_1+\alpha_2)}{m} & \text{if } k=0 \\ \frac{2\alpha_1}{m}, \frac{2\alpha_2}{m} & \text{if } k=\pi/a \end{cases}$$

## UV PHYSICS ACADEMY

SSP(4)

Crystal systems :

| Crystal | Relation between a, b and c; α, β and γ | Types of Bravais lattices | No. of Bravais lattices | Examples |
|---|---|---|---|---|
| Cubic | a = b = c<br>α = β = γ = 90° | SC, BCC FCC | 3 | Diamond, NaCl ZnS, $CaF_2$ |
| Tetragonal | a = b ≠ c<br>α = β = γ = 90° | SC, BCC | 2 | $NiSO_4$, $SnC_2$ |
| Orthorhombic | a ≠ b ≠ c<br>α = β = γ = 90° | SC, BCC, FCC, BACC | 4 | $KNO_3$, $BaSO_4$, $Mg_2$, $SiO_2$ |
| Trigonal (Rhombohedral) | a = b = c<br>α = β = γ ≠ 90° | SC | 1 | CaSO4, Calcite, quartz |
| Hexagonal | a = b ≠ c<br>α = β = 90°, γ = 120° | SC | 1 | AgI, SiO2 |
| Monoclinic | a ≠ b ≠ c<br>α = γ = 90°, β ≠ 90° | SC, BACC | 2 | $FeSO_4$, Borax |
| Triclinic | a ≠ b ≠ c<br>α ≠ β ≠ γ ≠ 90° | SC | 1 | $K_2Cr_2O_7$, $CuSO_4$ |

## UV PHYSICS ACADEMY

SSP(10)

Band theory and semi conductors :

Bloch's theorem $\psi(r+R) = \exp i(\bar{k}\cdot\bar{R})\,\psi(r)$

Electron velocity $v(k) = \frac{1}{\hbar}\nabla_k E(k)$

Effective mass(scalar) $m^* = \hbar^2\left[\frac{\partial^2 E(k)}{\partial k^2}\right]^{-1}$

Effective mass (Tensor) $m_{ij} = \hbar^2\left[\frac{\partial^2 E(k)}{\partial k_i \partial k_j}\right]^{-1}$

Energy $E(k) = \frac{\hbar^2 k^2}{2m_e}$; Mobility $\mu = \frac{v_d}{E} = \frac{eD}{k_B T}$

Net current density $\bar{J} = (n_e\mu_e + n_h\mu_h)e\bar{E}$ ;

for p – n junction diode current $I = I_0\left[\exp\left(\frac{eV}{\eta k_B T}\right) - 1\right]$

In forward bias, beyond barrier potential $I = I_0 e^{(eV/\eta kT)}$ ,

$I = -I_0$ in reverse bias, V is – ve, eV > kT

**Note : Effective mass is positive near the bottom of the band and negative near the top of the band.**

ψ = electron eigen state

$\bar{k}$ = bloch wave vector

r = position vector

R = Lattice vector

E(k) = energy band

v(k) = electron velocity for wave vector k

$k_i, k_j$ = components of k

μ = particle mobility

$v_d$ = mean drift velocity

D = diffusion coefficient

$n_e$ = electron number density

$n_h$ = hole number density

$\mu_e$ = electron mobility

$\mu_h$ = hole mobility

$I_0$ = saturation current

V = bias voltage

+ve for forward bias

– ve for reverse bias

## UV PHYSICS ACADEMY

**Dielectrics:** SSP(8)

**Clausius - Mosotti equation** $\frac{n\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$ ; $n = \frac{N_A\rho}{M}$

$\bar{E}_d = -\frac{N_d\bar{P}}{\varepsilon_0}$ ; $N_d$ =1/3 (sphere), 1 (thin slab ⊥ $\bar{P}$), 0 (thin slab || to $\bar{P}$)

$\bar{P} = \alpha\bar{E}_{loc}$ ; $\alpha = \frac{\bar{P}}{\bar{E}_{loc}}$

Fermigas (3D):

$g(E) = \frac{V}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}E^{1/2}$ , $g(E_F) = \frac{3}{2}\frac{nv}{E_F}$

Fermi wave number $K_F = (3n\pi^2)^{1/3}$ ; $v_F = \frac{\hbar k_F}{m_e}$

Fermi energy (T = 0), $E_F(0) = \frac{\hbar^2 k_F^2}{2m_e} = \frac{\hbar^2}{2m_e}(3n\pi^2)^{2/3}$ ;

$E_F(T) = E_F(0)\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_F(0)}\right)^2\right]$ ; $T_F = \frac{E_F}{k}$

$C_{v_e} = \frac{\pi^2}{3}g(E_F)k^2T = \frac{\pi^2 k^2}{2E_F}T$ $(T << T_F)$ ; Total kinetic energy (Temp. = 0) $U_0 = \frac{3}{5}nVE_F$

$N_A$ - Avagadro number,

ρ - volume charge density,

M - molecular weight,

α - molecular polarizability

$n = \frac{N_A\rho}{M}$ = number of molecules per unit volume

$\varepsilon_r$ - dielectric constant.

$\bar{E}_{loc}$ - local electrical field

$\bar{P}$ = Atomic polarizability

$\bar{E}_d$ = Depolarizing fields

g(E) = Electron density of states

$v_F$ = Fermi velocity

$T_F$= Fermi temperature

$C_V$ Electron heat capacity

E = electron energy

$E_F$ = Fermi energy

## UV PHYSICS ACADEMY

**SSP(13)**

Magnetisation $M = \frac{N\mu_m^2(\mu_0 H)}{3k_B T} = \frac{Ng^2\mu_B^2 j(j+1)}{3k_B T}\mu_0 H$

Susceptibility $\chi = \frac{M}{H} = \frac{Ng^2\mu_B^2\mu_0}{3k_B T} j(j+1)$

Super conductivity :

Isotopic effect $\sqrt{m}\cdot T_c = \text{const}$

Critical field $H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$

Specific heat of a solid $C = T\left(\frac{dH}{dT}\right)$

Specific heat difference $C_N - C_S = -T_c\left(\frac{dH_c}{dT}\right)^2 = -ve$

Specific heat of super conductor $(C_{el})_s = a\exp\left(-\frac{\Delta}{kT}\right)$

m = isotopic mass
$T_c$ = super conducting transition temperature
T = temperature
$C_N$, $C_S$ = specific heats of normal, super conductors respectively
$\Delta$ = energy gap (0.1 meV)
G = Gibb's potential
$S_N$, $S_s$ = entropies of normal, super conductors respectively
$H_c(0)$=critical field at T = 0 k
$(C_{lattice})_N$ = lattice specific heat of normal conductor
$(C_{lattice})_S$ = lattice specific heat of normal super conductor

Note : The properties of the lattice (crystal structure, Debye temp. etc) do not change when a material becomes super conductor

hence $(C_{lattice})_N = (C_{lattice})_S$ ; Entropy of a solid $S = -\left(\frac{\partial G}{\partial T}\right)_H$ ; $S_N - S_S = -H_c\frac{dH_c}{dT}$

## UV PHYSICS ACADEMY

**SSP(11)**

Effective density of states in the conduction band $N_c = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}$

Effective densityof states in the valence band $N_v = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2}$

Density of electrons in the conduction band at temperature T

$n = N_c\exp\left(-\frac{E_c - E_F}{kT}\right) = N_c\exp\left(-\frac{E_g}{2kT}\right)$; $E_c - E_F = \frac{E_g}{2}$

Density of holes in the valence band at temperature T

$p = N_v\exp\left(-\frac{E_g}{2kT}\right)$; $E_F - E_v = \frac{E_g}{2}$

Eg = energy gap of the semi conductor
h = Planck constant
$m_e^*$ = effective mass ele
$m_h^*$ = = effective mass of hole
k=Boltzman consatnt
$n_i$=intrinsic carrier concentration

Semiconductor equation: $np = n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^3 (m_e^* m_h^*)^{3/2}\exp\left(-\frac{E_g}{kT}\right)$; $E_c - E_v = E_g$

Intrinsic carrier concentration : $n_i = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_e^* m_h^*)^{3/4}\exp\left(-\frac{E_g}{2kT}\right)$

For intrinsic semiconductor : $E_F = \frac{E_c + E_v}{2} + \frac{kT}{2}\ln\frac{N_v}{N_c}$ ; $E_F = \frac{E_c + E_v}{2} + \frac{3kT}{4}\ln\frac{m_h^*}{m_e^*}$

## UV PHYSICS ACADEMY

### CLASSICAL MECHANICS

**CM(1)**

Degree of freedom (dof): f = 3N − k  N = number of particles, k = number of constraints

| Object | dof |
|---|---|
| 1. Rigid body | 6 |
| 2. Fly wheel | 1 |
| 3. Simple pendulum | 1 |
| 4. Spherical pendulum | 2 |
| 5. An ant moving on a non spherical baloon | 3 |
| 6. Particle inside a cone | 2 |
| 7. Compound pendulum | 1 |

**Constraints**

| Rheonomic | Scleronomic |
|---|---|
| explicit time dependence $f(r_n, t)$ | no explicit time dependence $f(r_n)$ |
| **Holonomic** | **Non holonomic** |
| integrable | Non integrable |
| $f(r_n, t) = 0$ | $f(r_n, t) \neq 0$, n = 1, 2,.. |

DeAlembert's principle : $\sum_{i=1}^{N}(F_i^{ext} - \dot{p}_i)\delta r_i = 0$

Least action principle : $S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, t)\,dt$

$\delta S = 0$

Eluer Lagrange equation : $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) = 0$

i = 1, 2, 3...s

Inexternal field : $L = \frac{1}{2}mv^2 - V(r, t) = T - V$

Generalised momenta : $p_i = \frac{\partial L}{\partial \dot{q}_i}$

$\delta r_i$ = virtual displacement
S = action; q = generalised co-ordinate
$\dot{q}$ = generalised velocities
L = Lagrangian

**Example of constraint:**

1) Two particle connected by a rod of variable length - **Holonomic rehenomic system**
2) A sphere rolling down from a top of fixed sphere - **scleronomic non-holonomic system**
3) A particle moving on a very long frictionless wire which rotates with constant angular speed about a horizontal axis - **reheonomic holonomic**

## UV PHYSICS ACADEMY

**CM(3)**

Hamiltonian $H = \sum_{i=1}^{n}\dot{q}_i p_i - L$ ; Hamiltons equation $\dot{q}_i = \frac{\partial H}{\partial p_i}$, $\dot{p}_i = -\frac{\partial H}{\partial q_i}$

Hamiltonian of a particle in an external field $H = \frac{1}{2}mv^2 + V(r, t) = T + V$

Hamiltonian in different co-ordinate system

Cartesian: $H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + V(x)$

Cylindrical: $H = \frac{1}{2m}\left(p_r^2 + \frac{p_\phi^2}{r^2} + p_z^2\right) + V(r)$

Spherical: $H = \frac{1}{2m}\left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2\sin^2\theta}\right) + V(r)$

Hamiltonian for some physical system ; Simple pendulum : $H = \frac{p_\theta^2}{2m\ell^2} - mg\ell\cos\theta$ ;

Canonical pendulum: $H = \frac{p_\phi^2}{2m\ell^2\sin^2\theta} - mg\ell\cos\theta$ ; Spherical pendulum $H = \frac{p_\theta^2}{2mr^2} + \frac{p_\phi^2}{2mr^2\sin^2\theta} + mgr\cos\theta$

A particle moving in electromagnetic field : $H = \frac{(p - qA)^2}{2m} + q\phi$ ,

Equation of motion : $\frac{df}{dt} = [f, H] + \frac{\partial f}{\partial t}$, $[f, H] = 0$ if $\frac{\partial f}{\partial t} = 0 \Rightarrow \frac{df}{dt} = 0$

A = vector potential
$\phi$ = scalar potential
PB = poisson bracket,
QC = Quantum commutator

**Poissons theorem :** f, g are integral of motion then [f, g] will be an integral of motion

i.e, $\frac{df}{dt} = \frac{dg}{dt} = 0 \Rightarrow \frac{d}{dt}[f, g] = 0$ ; **Jacobi Identity:** [f, [g, h]] + [g, [h, f]] + [h, [f, g]]=0

## UV PHYSICS ACADEMY

**CM(5)**

Small Oscillations:

Secular equation : $|V - \omega^2 T| = 0$, V = potential energy, T = Kinetic energy

| V | $q_1$ | $q_2$ | .... | $q_n$ |
|---|---|---|---|---|
| $q_1$ | $V_{11}$ | $V_{12}$ | .... | $V_{1n}$ |
| $q_2$ | $V_{21}$ | $V_{22}$ | .... | $V_{2n}$ |
| ⋮ | .... | .... | .... | .... |
| $q_n$ | $V_{n1}$ | $V_{n2}$ | .... | $V_{nn}$ |

| T | $\dot{q}_1$ | $\dot{q}_2$ | .... | $\dot{q}_n$ |
|---|---|---|---|---|
| $\dot{q}_1$ | $T_{11}$ | $T_{12}$ | .... | $T_{1n}$ |
| $\dot{q}_2$ | $T_{21}$ | $T_{22}$ | .... | $T_{2n}$ |
| ⋮ | .... | ..... | .... | .... |
| $\dot{q}_n$ | $T_{n1}$ | $T_{n2}$ | .... | $T_{nn}$ |

$a = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$

For normal modes eigen value equation is $|V - \omega^2 T|a = 0$

Frequencies of Normal modes:

Parallel pendula : $\omega_1 = \sqrt{\frac{g}{\ell}}$, $\omega_2 = \sqrt{\frac{g}{\ell} + \frac{2k}{m}}$ ; Double pendulum : $\omega = \sqrt{(2 \pm \sqrt{2})\frac{g}{\ell}}$

Linear triatomic molecule : $\omega_1 = 0, \omega_2 = \sqrt{\frac{k}{m}}, \omega_3 = \sqrt{\frac{2k}{M} + \frac{k}{m}}$

Three masses on a circle : $\omega = \sqrt{\frac{(2 \pm \sqrt{2})k}{m}}$, $\omega_1 = \sqrt{\frac{2k}{m}}$ ; Diatomic molecule $\omega = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$

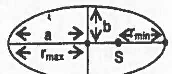
Coupled oscillator : $\omega = \sqrt{\omega_0^2 + 2\omega_c^2}$, $\omega^2 = \omega_0$

## UV PHYSICS ACADEMY

**CM(7)**

Different paths of motion of a particle

| Circle | Ellipse | Parabola | Hyperbola |
|---|---|---|---|
| $\varepsilon = 0, E = \frac{-k^2}{2mh^2}$ | $0 < \varepsilon < 1, E = -ve$ | $\varepsilon = 1, E = 0$ | $\varepsilon > 1, E = +ve$ |




Perihilion $r_{min} = \frac{L^2}{km(1+\varepsilon)}$ , Apehilian $r_{max} = \frac{L^2}{km(1-\varepsilon)}$

Semimajor axis : $a = \frac{-k}{2E} = \frac{k}{2|E|}$, $a = \frac{L^2}{mk(1-\varepsilon^2)}$ ; Semiminor axis : $b = \frac{L}{\sqrt{2m|E|}}$

**Rotating Frames:**

$\left(\frac{d\bar{r}}{dt}\right)_s = \left(\frac{d\bar{r}}{dt}\right)_{s'} + \bar{\omega}\times\bar{r}$ ; Coriolis force : $F_c = 2m(\bar{\omega}\times\bar{v})$ ; Euler force : $F_E = m\frac{d\bar{\omega}}{dt}\times\bar{r}$

Centripetal force : $F_{CN} = m\bar{\omega}\times(\bar{\omega}\times\bar{r})$ ; $\bar{L} = \bar{I}\bar{\omega}$ ; $T = \frac{1}{2}\bar{\omega}.\bar{I}.\bar{\omega}$

Rigid body dynamics:

Moment of inertia: For discrete mass distribution $I = \sum m_i r_i^2$, i = 1,2,3...

For continuous mass distribution $I = \int dm\,r^2$.

Parallel axis theorem : $I = I_0 + Md^2$ ; $K = \sqrt{\frac{I}{m}} = \sqrt{\frac{\sum mr^2}{\sum m}}$

Perpendicular axis theorem : $I_Z = I_X + I_Y$

S,S' = initial, reference frame
$\bar{I}$ = moment of inertia tensor
L = Angular momentum
T = Rotational kinetic energy
K = Radius of gyration
$I_0$ = moment of inertia passing through centre of gravity
d = distance from the centre of gravity to the point at which finding moment of inertia

## UV PHYSICS ACADEMY

**SSP(12)**
Magnetisim :
**Diamagnetism :**

Diamagnetic moment of anatom $\mu_m = -\frac{ze^2B}{6m_e}<r^2>$

Intrinsic electron magnetic moment $\mu = -\frac{e}{2m_e} g J$

Magnetization $M = N\mu_m$

Magnetic susceptibility $\chi_{dia} = \frac{M}{H} = -\frac{\mu_0 N z e^2}{6m_e}<r^2>$

Note : 1. Diamagnetic susceptibility is independent of temperature.
2. $\chi_{dia} \propto r^2$ outer electrons make larger contribution to the susceptibility

**Paramagnetism** Langevin's classical theory :

Potential energy of a magnetic dipole $U(magnetic) = -\vec{\mu}_m \cdot \vec{B}$

Magnetisation $M = N\mu_m L(x)$

$M = \frac{N\mu_m^2 B}{3kT}$ where $L(x) = \coth x - \frac{1}{x} \approx \frac{x}{3}$ (for $x \le 1$); $x = \frac{\mu_m B}{kT}$

**Curie's law :** $\chi = \frac{C}{T}$; $C = \frac{\mu_0 N \mu_m^2}{3k}$ ; **Curie - Weiss law :** $\chi = \frac{C}{(T-T_c)}$ $(T > T_c)$ ; $\chi \neq \infty$ and $\chi \neq -ve$

**Quantum theory of paramagnetism:** Magnetic moment $\mu_m = g\left(\frac{-e}{2m}\right)|J|$ , $\mu_m = g\mu_B\sqrt{j(j+1)}$

$<r^2>$ = mean square orbital radius (of all electrons)
z = atomic number
B = magnetic flux density
N = number of atoms per unit volume
L(x) = Langevin function
C = Curie constant
$T_c$ = Curie temperature
$\mu_B$ = Bohr magneton
H = magnetic field strength
j = total angular momentum quantum number
k = Boltzmann constant

## UV PHYSICS ACADEMY

**SSP(14)**
London equations:

$1^{st}$ equation $\frac{d\vec{J}_s}{dt} = \frac{n_s e^2}{m}\vec{E}$ ; $2^{nd}$ equation $\nabla \times \vec{J}_s = -\frac{n_s e^2}{m}\vec{B}$

London penatration depth $\lambda(T) = \lambda(0)\left[1-\left(\frac{T}{T_c}\right)^4\right]^{-1/2}$

Super conducting electron density $n_s = n_0\left[1-\frac{T^4}{T_c^4}\right]$

Order parameter $W = \frac{n_s}{n_0} = \left[1-\frac{T^4}{T_c^4}\right]$

Note: 1.Order parameter characterises the degree of order in the super conducting state.
2. The relation between the penatration depths and number of super electrons is inversly proportional and they are temperature dependent.

Coherence length $\xi_0 = \frac{2\hbar v_F}{\pi\Delta} \cong 10^{-6}m$ ; flux quantization $\phi = \frac{nhc}{2e} = n\phi_0$

**Josephson effect :**

D.C. current $I = I_c \sin\phi$ , $I_c = 2\pi r H_c(T)$ ;

A.C. current $I = I_c\sin(\omega t + \phi)$ ; $\omega = \frac{2eV}{\hbar}$ ; $\upsilon = \frac{2eV}{h}$

$J_s$ = current density of super electrons
$n_s$ = density of super electrons.
E = transient electric field arises with in a super conductor which freely accelerates super electrons
B = magnetic field
$\lambda(0)$ = penatration depth at T = 0 k
$T_c$ = critical or transition temperature
r = radius of super conducting current carrying ring
$\phi$ = phase difference b/w wave functions of cooper pair. $\phi_0 = hc/2e = 2.07 \times 10^{-7}$ gauss - $cm^2$ (fluxoid or flux quantum)
$v_F$ = electron velocity at the fermi surface
$\upsilon$ = frequency of super electrons

## UV PHYSICS ACADEMY

**CM(4)**

Poisson Bracket Properties : $[x, p]_{PB} = \frac{1}{i\hbar}[x, p]_{QC}$

$[f, f] = [f, c] = 0$ ; $[f, g+h] = [f, g] + [f, h]$ ; $[f, gh] = [f, g]h + g[f, h]$ ; $[f, g] = -[g, f]$

Canonical transformation:

Generating function $H' - H = \frac{\partial F}{\partial t}$ Generating function

$F_1(q_i, Q_i, t)$ : $p_i = \frac{\partial F_1}{\partial q_i}, P_i = \frac{-\partial F_1}{\partial Q_i}$

$F_2(q_i, P_i, t)$ : $p_i = \frac{\partial F_2}{\partial q_i}, Q_i = \frac{\partial F_2}{\partial P_i}$

$F_3(p_i, Q_i, t)$ : $q_i = \frac{-\partial F_3}{\partial p_i}, P_i = \frac{-\partial F_3}{\partial Q_i}$

$F_4(p_i, P_i, t)$ : $q_i = \frac{-\partial F_4}{\partial p_i}, Q_i = \frac{\partial F_4}{\partial P_i}$

**Condition for Canonical transformation:**

1) Exact differentiable : $\Sigma(p_i dq_i - P_i dQ_i) = dF$
2) Invariance of poisson bracket : $[Q, Q] = [P, P] = 0$, $[Q, P] = 1$
3) $Q = f(q, p), P = f(q, p)$ $\begin{vmatrix} \frac{\partial Q}{\partial q} & \frac{\partial Q}{\partial p} \\ \frac{\partial P}{\partial q} & \frac{\partial P}{\partial p} \end{vmatrix} = 1$ $q = f(Q, P), p = f(Q, P)$ $\begin{vmatrix} \frac{\partial q}{\partial Q} & \frac{\partial q}{\partial P} \\ \frac{\partial p}{\partial Q} & \frac{\partial p}{\partial P} \end{vmatrix} = 1$
4. Bilinear condition : $\sum_k(\delta p_k dq_k - \delta q_k dp_k) = \sum_k(\delta P_k dQ_k - \delta Q_k dP_k)$

## UV PHYSICS ACADEMY

**CM(2)**

Lagrangian in different co-ordinate system

Cartesian $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V$

Cylindrical $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2 + \dot{z}^2) - V(r)$

Spherical $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) - V(r)$

Lagrangian of some physical systems: Conical pendulum : $L = \frac{1}{2}m\ell^2\sin^2\theta\dot{\phi}^2 + mg\ell\cos\theta$

Simple pendulum : $L = \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell(1-\cos\theta)$ ; Compound pendulum : $L = \frac{1}{2}I\dot{\theta}^2 + mg\ell\cos\theta$

Spherical pendulum : $L = \frac{1}{2}m(r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) - mgr\cos\theta$

yo-yo with moving support : $L = \frac{1}{2}m(\dot{z}^2 - 2a\dot{z}\dot{\theta} + \frac{3}{2}a^2\dot{\theta}^2) - mg(z - a\theta)$

Charged particle in an electromagnetic field : $L = \frac{1}{2}mv^2 + \frac{q}{c}\vec{v}\cdot\vec{A} - q\phi$

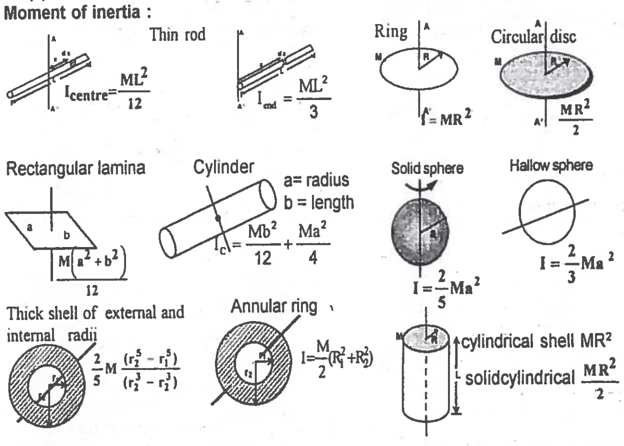
A particle moving in a central force : $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{k}{r}$

A bead sliding on a firctionless wire in the shape of a cycloid : $L = ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$

Poissons bracket : $[f, g] = \sum_i\left[\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}\right]$ ; $[q_i, g] = \frac{\partial g}{\partial p_i}$, $[p_i, g] = -\frac{\partial g}{\partial q_i}$

## UV PHYSICS ACADEMY

**CM(8)**
**Moment of inertia :**

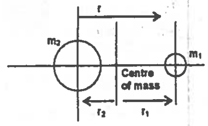



Euler Equations

$\tau_1 = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2)$ $\tau_2 = I_2\dot{\omega}_2 + \omega_1\omega_3(I_1 - I_3)$ $\tau_3 = I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1)$

## UV PHYSICS ACADEMY

**CM(6)** **Central force**




$\mu = \frac{m_1 m_2}{m_1 + m_2}$, $r_1 = \frac{m_2 r}{m_1 + m_2}$, $r_2 = \frac{-m_1 r}{m_1 + m_2}$ ; $I = \mu|r|^2$, $J = \mu r \times \dot{r}$

Gravitationally bound orbital motion : $h = r^2\dot{\theta} = \frac{L}{m}$ ,

areal velocity $\frac{dA}{dt} = \frac{1}{2}r^2\dot{\theta} = \frac{h}{2}$ = constant

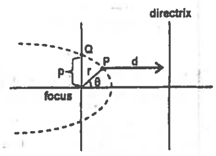
Equation of motion for a particle in a central field : $\frac{d^2u}{d\theta^2} + u = \frac{-f\left(\frac{1}{u}\right)}{mh^2u^2}$, $u = \frac{1}{r}$

Energy conservation $\left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{2(E-V)}{mh^2}$, $V = -\int F(r)dr$

Virial theorem : $F \propto r^n$ or $V \propto r^{n+1}$ ; $<K> = \frac{n+1}{2}<V>$, $<K> = \frac{1}{2}<r\frac{\partial V}{\partial r}>$

$u = \frac{1}{r} = \frac{k}{mh^2}\left[1 + \sqrt{1 + \frac{2Emh^2}{k^2}}\cos\theta\right]$ ; $\frac{1}{r} = \frac{1}{p}(1 + \varepsilon\cos\theta)$ ; $p = \frac{mh^2}{k}$, $\varepsilon = \sqrt{1 + \frac{2Emh^2}{k^2}}$

$\mu$ = reduced mass of two interacting bodies; F = Force
$r_1$, $r_2$ = distance from centre of mass; I = moment of inertiaJ = angular momentum; V = potential energy;
T = kinetic energy; h = angular momentum per unit mass
<K> = average value of kinetic energy; $\varepsilon$ = ecentricity
<V> = average valueof potential energy , n = integer



## UV PHYSICS ACADEMY

**CM(9)**

### Moment of inertia tensor

For discrete mass distribution

$$I_{ij} = \sum_{\alpha} m_{\alpha}(r_{\alpha}^2\delta_{ij} - x_i x_j)$$

For continuous mass distribution

$$I = \int_V \rho(r)(r^2\delta_{ij} - x_i x_j)dV$$

Principle moment of inertia tensor $I = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$

### Symmetries

**Asymmetric top**

$I_1 \neq I_2 \neq I_3$

$\tau_1 = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2)$

$\tau_2 = I_2\dot{\omega}_2 + \omega_1\omega_3(I_1 - I_3)$

$\tau_3 = I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1)$

**Symmetric top**

$I_1 = I_2 \neq I_3$

$\tau_1 = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2)$

$\tau_2 = I_2\dot{\omega}_2 + \omega_1\omega_3(I_1 - I_3)$

$\tau_3 = I_3\dot{\omega}_3$

**Spherical top**

$I_1 = I_2 = I_3$

$\tau_1 = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2)$

$\tau_2 = I_2\dot{\omega}_2 + \omega_1\omega_3(I_1 - I_3)$

$\tau_3 = I_3\dot{\omega}_3$

**Rotar**

$I_1 = I_2 \neq I_3$, but $I_3 = 0$

$\tau_1 = I_1\dot{\omega}_1 + \omega_2\omega_3(-I_2)$

$\tau_2 = I_2\dot{\omega}_2 + \omega_1\omega_3(I_1)$

Cone : $\rho = \dfrac{M}{\frac{1}{3}\pi R^2 h}$, $I_{xx} = I_{yy} = \dfrac{Mh^2}{10} + \dfrac{3}{20}MR^2$, $I_{zz} = \dfrac{3}{10}MR^2$

Hoop : $M = \pi R^2 h\rho$, $I_{xx} = I_{yy} = \dfrac{1}{12}\pi h R^2\rho(h^2 + 3R^2)$, $I_{zz} = \dfrac{1}{2}\pi h R^4\rho$

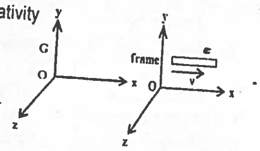
## UV PHYSICS ACADEMY

**CM(11)**

### Special theory of relativity

Time dilation : $t_G = \dfrac{t_f}{\sqrt{1-(v^2/c^2)}} \Rightarrow t_G = \gamma t_f$

Length contraction : $S^I$ frame is moving with velocity v w.r.t G. The rod is kept at rest in force $\ell_G = \dfrac{\ell_f}{\gamma}$




**Lorentz transformation:** $x^I = \gamma(x - vt)$, $y^I = y$, $z^I = z$, $t^I = \gamma\left(t - \dfrac{vx}{c^2}\right)$

for $\beta = v/c$, $x^I = \gamma(x - \beta ct)$, $y^I = y$, $z^I = z$, $ct^I = \gamma(ct - \beta c)$

$$\begin{bmatrix} ct^I \\ x^I \\ y^I \\ z^I \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

Eigen values of Lorentz matrix are 1, 1, $\gamma(1-\beta)$, $\gamma(1+\beta)$

$S^2 = c^2t^2 - \gamma^2$

- $S^2 > 0$: $c^2t^2 > \gamma^2$, time like, $\left|\dfrac{\gamma}{t}\right| < c$
- $S^2 < 0$: $c^2t^2 < \gamma^2$, space like, $\left|\dfrac{\gamma}{t}\right| > c$
- $S^2 = 0$: $c^2t^2 = \gamma^2$, light like, $\left|\dfrac{\gamma}{t}\right| = c$

$t_G$ = time w.r.t ground
$t_f$ = time w.r.t. force

$\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$; $\gamma > 1$

v = velocity of frame
$\ell_G$ = length of the object w.r.t ground.
$\ell_f$ = length of object in moving frame
$\gamma$ = lorentz factor

## UV PHYSICS ACADEMY

**CM(13)**

Dynamics : $E = mc^2$

- Kinematic energy $(m - m_0)c^2$
- Rest mass $m_0$, $m_0c^2$

$E = mc^2$ = total energy of particle
$E_0 = m_0c^2$ = rest mass energy

$m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 = ict$
$J_\mu = (j, ic\rho) = (J_1, J_2, J_3, J_4)$

Four vectors :

$$\begin{bmatrix} x_1^I \\ x_2^I \\ x_3^I \\ x_4^I \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$x_1^I = \gamma(x + i\beta x_4)$
$x_2^I = x_2$, $x_3^I = x_3$
$x_4^I = \gamma(x_4 - i\beta x_1)$

Charge density four vector : $J_\mu = \rho\dfrac{dx_\mu}{dt}$, covariant form : $\dfrac{\partial J_\mu}{\partial x_\mu} = 0$

Position : $\bar{x}_\mu = (\bar{r}, ict)$, $x_\mu = (x_1, x_2, x_3, x_4 = ict)$ Momentum $\bar{p}_\mu = m_0\bar{v}_\mu = (m\bar{u}, mic)$, $\bar{p}_\mu = (\bar{p}, iE/c)$

$$\bar{v}_\mu = \left(\bar{u}/\sqrt{1-(u^2/c^2)}, ic/\sqrt{1-(u^2/c^2)}\right)$$

$$\bar{F}_\mu = \frac{d\bar{p}_\mu}{dt} = \left(\bar{F}/\sqrt{1-(u^2/c^2)}, \frac{i}{c}\bar{F}.\bar{u}/\sqrt{1-(u^2/c^2)}\right)$$

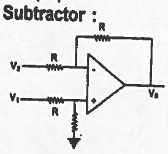
$$\bar{a}_u = \left[\frac{\bar{a}}{1 - u^2/c^2} + \frac{u(\bar{u}.\bar{a})}{c^2(1-u^2/c^2)}, \frac{i(\bar{u}.\bar{a})}{c(1-u^2/c^2)^2}\right]$$

## UV PHYSICS ACADEMY

**ELE(14)**

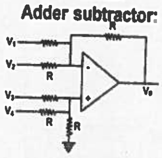
### OPAMP

**Subtractor :**




$V_0 = V_1 - V_2$

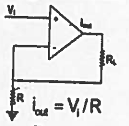
**Adder subtractor:**




$V_0 = (V_3 + V_4) - (V_2 + V_1)$

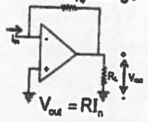
**Converters**

Voltage to Current




$i_{out} = V_i/R$
$Z_{in} = \infty$

Current to Voltage




$V_{out} = RI_n$
$Z_{out} = 0$

**Phase shift oscillator:** $\beta$ = feedback ratio = 1/29, $f_r = 1/(2\pi\sqrt{6}RC)$

**Colpitt's oscillator :** $f_r = 1/(2\pi\sqrt{LC_T})$, $C_T = C_1C_2/(C_1 + C_2)$

**Hartley oscillator :** $f_r = 1/(2\pi\sqrt{L_TC})$, $L_T = L_1 + L_2 + 2M$

M=mutual inductance; $\beta = L_2/L_1$
H(jw)= voltage transfer function

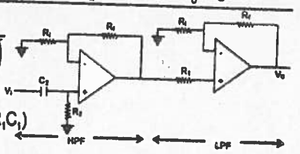
**Low pass filter:** $H(j\omega) = A_0/\sqrt{1 + (\omega/\omega_H)^2}$
$\omega_h = 1/(RC)$, $f_C = 1/(2\pi RC)$, $A_0 = 1 + (R_f/R_1)$

**High pass filter :** $H(j\omega) = A_0/\sqrt{1 + (\omega_L/\omega)^2}$
$f_L = 1/(2\pi RC)$, $\omega_L = 1/(RC)$; $A_0$ = gain

**Band pass filter :** $H(j\omega) = \dfrac{A_0(f/f_L)}{\sqrt{(1 + (f/f_L)^2)(1 + (f/f_H)^2)}}$

$A_0 = A_{01}A_{02}$ ; $A_{01}(HPF) = 1 + (R_f/R_1)$ ;

$A_{02}(LPF) = 1 + (R_f/R_1)$ ; $f_L = 1/(2\pi R_2C_2)$, $f_H = 1/(2\pi R_1C_1)$




## UV PHYSICS ACADEMY

**ELE(12)**

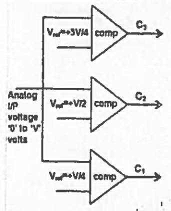
% Resoultion = $\dfrac{100}{2^n - 1}$%, Voltage conversion resoultion = $\dfrac{V_{ref}}{2^n}$

Resoution = the weight of LSB (or)

$$K = \frac{V_{FS}}{\text{total no. of steps}} = \frac{\text{Analog full scale voltage}(V_{FS})}{2^n - 1}$$

### A/D Convertor (ADC)

Simultaneous ADC




| I/P voltage | C1 | C2 | C3 |
|---|---|---|---|
| 0 to V/4 | L | L | L |
| V/4 to V/2 | H | L | L |
| V/2 to 3V/4 | H | H | L |
| 3V/4 to V | H | H | H |

Counter of RAMP type ADC : conversion time = $\dfrac{V_{in}2^nT_C}{V_{ref}}$

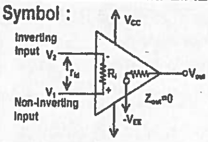
n → no of bits in digital word, $T_C$ = time period of clock pulse. Max conversion time = $t_{cm} = (2^n - 1)$ clock cycle

Average conversion time = $\dfrac{t_{cm}}{2} = 2^{n-1}$

c) Dual slope type ADC = $V_{in} = NV_{ref}/2^{n-1}$

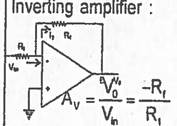
### OPERATIONAL AMPLIFIER

Symbol :

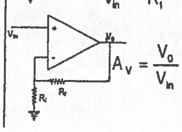



Open loop voltage gain $(A_v) = \infty$
Input impedence = $\infty$
Output impedence = 0
Bandwidth = $\infty$
Zero offset i.e, $V_{out} = 0$ where $V_1 - V_2 = 0$

Inverting amplifier :




$A_V = \dfrac{V_0}{V_{in}} = \dfrac{-R_f}{R_1}$



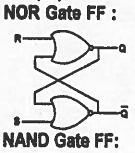

$A_V = \dfrac{V_0}{V_{in}} = 1 + \dfrac{R_f}{R_1}$

## UV PHYSICS ACADEMY

**ELE(10)**

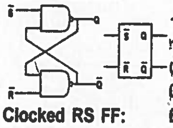
### FLIP – FLOP (FF)

**NOR Gate FF :**



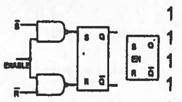

| R | S | Q | Action |
|---|---|---|---|
| 0 | 0 | last state | no change |
| 0 | 1 | 1 | Set |
| 1 | 0 | 0 | Reset |
| 1 | 1 | x | Forbidden |

**NAND Gate FF:**



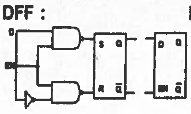

| $\bar{R}$ | $\bar{S}$ | Q |
|---|---|---|
| 1 | 1 | last state |
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | forbidden |

**Clocked RS FF:**



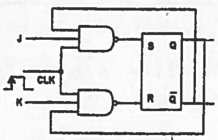

| EN | S | R | $Q_{n+1}$ |
|---|---|---|---|
| 1 | 0 | 0 | $Q_n$(no change) |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | Illegal |
| 0 | x | x | $Q_n$(no change) |

**DFF :**



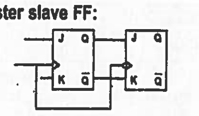

| EN | D | $Q_{n+1}$ |
|---|---|---|
| 0 | x | $Q_n$(last state) |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**JK FF :**




| CLK | J | K | $Q_{n+1}$ | Action |
|---|---|---|---|---|
| ↑ | 0 | 0 | $Q_n$ | no change |
| ↑ | 0 | 1 | 0 | ReSet |
| ↑ | 1 | 0 | 1 | set |
| ↑ | 1 | 1 | $\bar{Q}_n$(toggle) | toggle |

**JK Master slave FF:**




| CLK | J | K | $Q_{n+1}$ | Action |
|---|---|---|---|---|
| ↑ | 0 | 0 | $Q_n$ | no change |
| ↑ | 0 | 1 | 0 | ReSet |
| ↑ | 1 | 0 | 1 | set |
| ↑ | 1 | 1 | $\bar{Q}_n$(toggle) | toggle |

## UV PHYSICS ACADEMY

**CM(12)**

Relativistic addition of velocities: $u_G = \dfrac{u_F + v}{1+(u_F v/c^2)}$

Relativistic addition of velocities

- Frame, velocity both are moving away from the ground: $u_G = \dfrac{u_F + v}{1+(u_F v/c^2)}$
- Frame is moving away from the the ground but particle is moving towards ground in that frame: $u_G = \dfrac{-u_F + v}{1-(u_F v/c^2)}$

Velocity components

- Parallel: $u_x' = \dfrac{u_x - v}{(1-(u_x v/c^2))}$
- Perpendicular: $u_y' = \dfrac{u_y}{\gamma(1-(u_x v/c^2))}$, $u_z' = \dfrac{u_z}{\gamma(1-(u_x v/c^2))}$

> $u_G, u_F$ = velocity w.r.t ground and frame
> v = velocity of moving frame
> $u_x, u_x'$ = velocity of object in rest frame, in moving frame respectively
> f, f'= frequency in rest frame , moving frame respectively
> $\beta = \dfrac{v}{c}$

Longitudinal Doppler effect $\theta = 0^0$ $\omega = \omega'\sqrt{(1+\beta)/(1-\beta)}$

- frame is moving away from the observer $f' < f$ (Red Shift)
- frame is moving towards the observer $f' > f$ (Blue Shift)

$$\omega = \frac{\omega'\sqrt{1-\beta^2}}{1-\beta\cos\theta}$$

Transverse Doppler effect $\theta = 90^0$ $\omega = \omega'\sqrt{(1-\beta^2)}$

## UV PHYSICS ACADEMY

**CM(10)**

Cylinder : $\rho = \dfrac{M}{\pi R^2 h}$, $I_{xx} = I_{yy} = \dfrac{Mh^2}{12} + \dfrac{1}{4}MR^2$, $I_{zz} = \dfrac{MR^2}{2}$

Ellipse : $\sigma = \dfrac{M}{\pi ab}$, $I_{xx} = \dfrac{Ma^2}{4}$, $I_{yy} = \dfrac{Mb^2}{4}$

Hoop: $M = \pi R^2 h\rho$, $I_{xx} = I_{yy} = \dfrac{1}{12}\pi h R^2 \rho(h^2 + 3R^2)$, $I_{zz} = \dfrac{1}{2}\pi h R^4 \rho$ ;Rectangle: $I_{xx} = \dfrac{Ma^2}{3}$, $I_{yy} = \dfrac{Mb^2}{3}$

Ellipsoide : $I_c$ = moment of inertia about the minor axis c = $\dfrac{M}{5}(a^2+b^2)$

$I_a$ = moment of inertia about the minor axis a = $\dfrac{M}{5}(b^2+c^2)$

$I_b$ = moment of inertia about the minor axis a = $\dfrac{M}{5}(a^2+c^2)$

Rectangular parallelopiped $\rho = \dfrac{M}{8abc}$, $I_{xx} = \dfrac{(b^2+c^2)M}{3}$, $I_{yy} = \dfrac{(a^2+c^2)M}{3}$, $I_{zz} = \dfrac{(a^2+b^2)M}{3}$

**Routh's rule:**

| n | body |
|---|---|
| 3 | Rectangle |
| 3 | Rectangular parallelopiped |
| 4 | Ellipse |
| 5 | Ellipsoide |

Principle moment of inertia for simple bodies $I = \dfrac{MS^2}{n}$

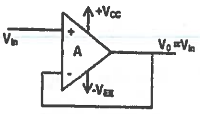
$S^2$ = sum of squares of the semimajor axes of the other principle moments.

n = a small positive integer

## UV PHYSICS ACADEMY

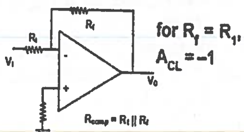
**ELE(13)**

**Voltage follower :**



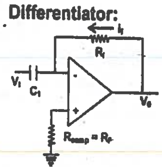

**Operational Amplifier**

**Scale changer / inverter**




for $R_f = R_1$, $A_{CL} = -1$

**Differentiator:**

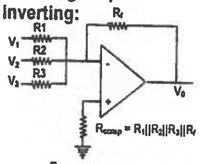



$V_0 = -R_F C_1 \dfrac{dV_i}{dt}$

gain A $= \omega R_F C_1$

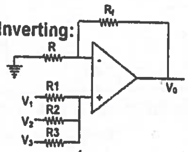
**Summing Amplifier:**

**Inverting:**



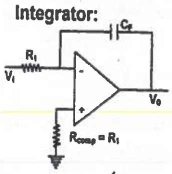

$V_0 = -\left[\dfrac{R_f}{R_1}V_1 + \dfrac{R_f}{R_2}V_2 + \dfrac{R_f}{R_3}V_3\right]$

**Non-Inverting:**




$V_0 = \left(1+\dfrac{R_f}{R}\right)\dfrac{\left(\dfrac{V_1}{R_1}+\dfrac{V_2}{R_2}+\dfrac{V_3}{R_3}\right)}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}$

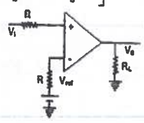
**Integrator:**



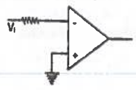

$V_0 = \dfrac{-1}{R_1 C_F}\int V_i dt$

gain A $= \dfrac{1}{\omega R_1 C_F}$

**Non-inverting comparator:**




**Inverting comparator**




## UV PHYSICS ACADEMY

**CM(14)**

acceleration : $\bar{a} = \dfrac{d\bar{v}_\mu}{dt} = \left[\dfrac{c^2}{c^2-u^2}\bar{a} + \bar{u}\cdot\dfrac{c^2(\bar{u}\cdot\bar{a})}{(c^2-u^2)^2}, \dfrac{ic^3(\bar{u}\cdot\bar{a})}{(c^2-u^2)^2}\right]$

Four vector potential : $A_\mu = (A_1, A_2, A_3, A_4) = (A, i\phi/c)$ $\square^2 = \nabla^2 - \dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}$

Coulomb gauge condition : $\bar{\nabla}\cdot\bar{A} = 0$

$$\begin{bmatrix} A_1' \\ A_2' \\ A_3' \\ A_4' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ i\phi/c \end{bmatrix}$$

Electromagnetic field four-tensor

$$F\mu\upsilon = \begin{bmatrix} 0 & B_z & -B_y & -iE_x/c \\ -B_z & 0 & B_x & -iE_y/c \\ B_y & -B_x & 0 & -iE_z/c \\ iE_x/c & iE_y/c & iE_z/c & 0 \end{bmatrix}$$

$F\mu\upsilon = \dfrac{\partial A_\upsilon}{\partial x_\mu} - \dfrac{\partial A_\mu}{\partial x_\upsilon}$; $F_{\mu\upsilon} = -F_{\upsilon\mu}$ & $F_{\mu\mu} = F_{\upsilon\upsilon} = 0$

Mechanical vs Electrical oscillator

| Mechanical | m (mass) | b (dissipative coefficient) | k (force const.) | F(t) (external force) | T $\frac{1}{2}m\dot{x}^2$ | U $\frac{1}{2}kx^2$ | F $\frac{1}{2}b\dot{x}^2$ (dissipative force) |
|---|---|---|---|---|---|---|---|
| electrical circuit (q) | L (inductance) | R (Resistance) | 1/C | E(t) (e.m.f) | $\frac{1}{2}L\dot{q}^2$ | $\frac{1}{2}\frac{q^2}{C}$ | $\frac{1}{2}R\dot{q}^2$ |

## UV PHYSICS ACADEMY

**ELE(9)**

**Boolean Laws**

$A+0=A$; $A+\bar{A}=1$; $\bar{\bar{A}}=A$; $A+B=B+A$; $A+(B+C)=(A+B)+C$; $A+1=1$;
$A(B+C)=AB+AC$; $\overline{A+B}=\overline{AB}$; $A+AB=A$; $A+\bar{A}B=A+B$; $A\cdot 0=0$; $A\cdot\bar{A}=0$;
$AB=BA$; $A(BC)=(AB)C$; $A+BC=(A+B)(A+C)$; $A\cdot 1=A$; $\overline{AB}=\bar{A}+\bar{B}$; $A(A+B)=A$

**DeMorgan's Law :** $\overline{A+B}=\bar{A}\bar{B}$, $\overline{AB}=\bar{A}+\bar{B}$

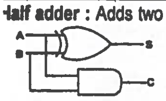
**Karnaugh map :**

| | $\bar{B}$ | B |
|---|---|---|
| $\bar{A}$ | | |
| A | | |

| | $\bar{B}$ | B |
|---|---|---|
| $\bar{A}\bar{B}$ | | |
| $\bar{A}B$ | | |
| AB | | |
| $A\bar{B}$ | | |

| | $\bar{C}\bar{D}$ | $\bar{C}D$ | CD | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | | | | |
| $\bar{A}B$ | | | | |
| AB | | | | |
| $A\bar{B}$ | | | | |

**Half adder :** Adds two binary digits at a time

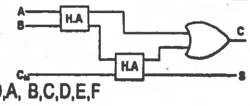



$S = \bar{A}B + A\bar{B}$

| Input | | Output | |
|---|---|---|---|
| A | B | sum(S) | carry (C) |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

**Full adder :** Adds three digits at a time

| Base | Number System | Base digit use in number system |
|---|---|---|
| 2 | Binary | 0, 1 |
| 3 | Teritary | 0, 1, 2 |
| 8 | Octal | 0, 1, 2, 3, 4, 5, 6, 7 |
| 10 | Decimal | 0, 1, 2, 3, 4, 5, 6, 7,8,9 |
| 16 | Hexadecimal | 0, 1, 2, 3, 4, 5, 6, 7,8,9,A, B,C,D,E,F |




## UV PHYSICS ACADEMY

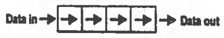
**ELE(11)**

**Registers**

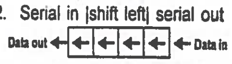
A group of FF's used to store a binary number.

**Shift registers :**

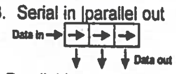
1. Serial in |shift right| serial out



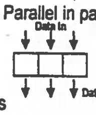

2. Serial in |shift left| serial out



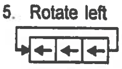

3. Serial in |parallel out



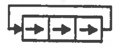

4. Parallel in parallel out




5. Rotate left



6. Rotate right

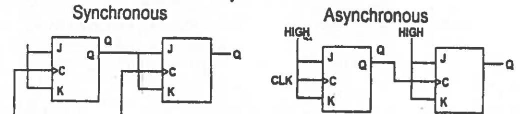


**Counters :**

Used to count the no. of clock cycles

Synchronous

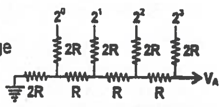
Asynchronous




**D/A Converter (DAC)**

**Binary Ladder :**

| Bit position | Binary weight | output voltage |
|---|---|---|
| MSB | 1/2 | V/2 |
| 2nd MSB | 1/4 | V/4 |
| ⋮ | ⋮ | ⋮ |
| Nth MSB | $1/2^N$ | $V/2^N$ |




$V_A = \dfrac{V_0 2^0 + V_1 2^1 + \ldots + V_{n-1}2^{n-1}}{2^n}$

# UV PHYSICS ACADEMY

## DIGITAL ELECTRONICS

ELE(8)

**Logic Gates :**

**OR Gate** ($Y = A + B$)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**AND Gate :** ($Y = AB$)

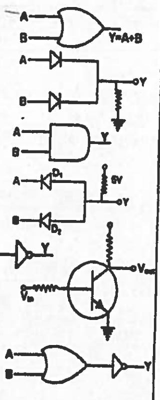
| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**NOT Gate :** ($Y = \overline{A}$)

| A | Y |
|---|---|
| 0 | 1 |
| 1 | 0 |

**NOR Gate :** ($Y = \overline{A+B}$)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |




**NAND Gate:** ($Y = \overline{AB}$)

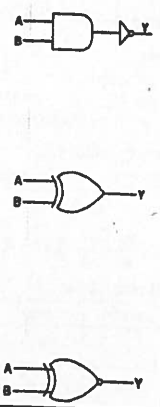
| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**Exclusive ORGate :** ($Y = \overline{A}B + A\overline{B}$)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**Exclusive NORGate :** ($Y = AB + \overline{A}\,\overline{B}$)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |




Equvalence of positive and negative logic : Positive OR ↔ negative AND, Positive AND ↔ negative OR, Positive NOR ↔ negative NAND, Positive NAND ↔ Negative NOR

# UV PHYSICS ACADEMY

ELE(6)

Alternating current : (current changes in magnitude and direction sinusoidally)

$$i_{avg} = i_{dc} = \frac{\int_0^{T/2} i_0 \sin\omega t}{\int_0^{T/2} dt} = \frac{2i_0}{\pi}$$

$i_{avg}$ = average current; $i_{dc}$ = D.C current; $i_0$ = maximum current; T = time period; z = impedence = R(for pure resistance), reactance ($x_L$ for pure inductor (L), $x_C$ for pure capacitor (C)) ; $\phi$ = phase difference b/w i and E.

Note : Average value of A.C over one complete cycle is zero.

| parametre | A.C. through | | | | | |
|---|---|---|---|---|---|---|
| | R only | L only | C only | R and L | R and C | L and C |
| i | $\frac{E_0}{R}$ | $\frac{E_0}{x_L}$ | $\frac{E_0}{x_C}$ | $\frac{E_0}{z}\sin(\omega t-\phi)$ | $\frac{E_0}{z}\sin(\omega t+\phi)$ | $\frac{E_0}{z}\sin(\omega t-(\pi/2))$ |
| z | R | $x_L = \omega L$ | $x_C = \frac{1}{\omega C}$ | $Z = \sqrt{R^2 + x_L^2}$ | $Z = \sqrt{R^2 + x_C^2}$ | $z = x_L - x_C$ |
| $\phi$ | i & E in phase(0°) | i lags E by 90° | i leads E by 90° | i lags E by $\tan^{-1}\left(\frac{\omega L}{R}\right)$ | i leads E by $\tan^{-1}\left(\frac{1}{\omega CR}\right)$ | i lags E by 90° |

Series LCR circuit : $L\frac{di}{dt} + iR + \frac{q}{C} = E_0 \sin\omega t$ ; $i = i_0 \sin(\omega t - \phi)$ ; $i_0 = (E_0/z)$

Impedence $z = \sqrt{R^2 + (x_L - x_C)^2}$

# UV PHYSICS ACADEMY

ELE(4)

**Transformer :** $\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1}{k}$ ;

Step up transformer: $N_2 > N_1$, $V_2 > V_1$, $I_2 < I_1$

Step down transformer: $N_2 < N_1$, $V_2 < V_1$, $I_2 > I_1$

**Rectifires :**

| Parameter | HWR | Centretaped FWR |
|---|---|---|
| D.C current ($i_{dc}$) | $\frac{i_0}{\pi}$ | $\frac{2i_0}{\pi}$ |
| D.C. voltage ($V_{dc}$) | $\frac{V_0}{\pi}\frac{R_L}{(R_L+R_f)} = \frac{V_0}{\pi}$ $(R_L > R_f)$ | $\frac{2V_0}{\pi}$ $(R_L > R_f)$ |
| RMS current ($i_{rms}$) | $\frac{i_0}{2}$ | $\frac{i_0}{\sqrt{2}}$ |
| RMS voltage ($V_{rms}$) | $\frac{V_0}{2}$ | $\frac{V_0}{\sqrt{2}}$ |
| A.C power ($P_{ac}$) | $(i_0^2/4)(R_L+R_f)$ | $(i_0^2/2)(R_L+R_f)$ |
| D.C power ($P_{dc}$) | $\frac{i_0^2}{\pi^2}R_L$ | $\frac{4i_0^2}{\pi^2}R_L$ |
| Efficiency $\eta = \left(\frac{P_{dc}}{P_{ac}}\right)$ $(R_L > R_f)$ | 40.6% | 81.2% |
| Peak inverse voltage | $V_0$ | $2V_0$ |

for full wave bridge rectifier PIV $=V_0$, remaining all parameters are same as FWR.

# UV PHYSICS ACADEMY

ELE(2)

**Comparision b/w CB, CE and CC configurations:**

| Parameter | CB | CE | CC |
|---|---|---|---|
| Input resistance ($r_i$) | very low (20 Ω) | moderate (1 kΩ) | very high (1 MΩ) |
| Output resistance ($r_0$) | very high (1M Ω) | high (10 kΩ) | very low (20 Ω) |
| current gain | $\alpha < 1$ | high ($\beta \approx 100$) | highest ($\gamma = \beta + 1$) |
| voltage gain | large ($\approx 1500$) | very large (1500) | $\approx 1$ |
| power gain | high (30 dB) | very high (40 dB) | low (10 to 20 dB) |
| phase reversal | 0° | 180° | 0° |
| Leakage current | very small ($\approx \mu A$) | very large ($\approx 100\,\mu A$) | very large ($\approx 100\,\mu A$) |

Note : 1. Most of the practical amplifiers work in CE mode because of its large current and voltage gains and hence the power gain.

2. Common collector (CC) circuit is also called as emitter follower circuit.

**Base resistor (or) fixed bias:**

$V_{CC} = V_{BE} + I_B R_B$

$I_B = \frac{V_{CC} - V_{BE}}{R_B}$

$V_{CC}$ and $R_B$ are fixed and $V_{BE}$ is small

$V_{CC}$ = supply voltage
$V_{BE}$ = base - emitter voltage
$I_B$ = base current
$R_B$ = base resistance

EMT

# UV PHYSICS ACADEMY

## ELECTROMAGNETIC THEORY

EMT(1)

Conversion from S.I to Gaussian units

| | | |
|---|---|---|
| $\varepsilon_0 \to \frac{1}{4\pi}$ | $\mu_0 \to \frac{4\pi}{c^2}$ | $B \to \frac{B}{c}$ |
| $\chi_E \to 4\pi\chi_E$ | $\chi_H \to 4\pi\chi_H$ | $H \to \frac{cH}{(4\pi)}$ |
| $A \to \frac{A}{c}$ | $M \to cM$ | $D \to \frac{D}{4\pi}$ |

λ = linear charge density
σ = surface charge density
ρ = volume charge density

Note : The quantities ρ, J, E, ϕ, σ, P, $\varepsilon_r$ & $\mu_r$ all are unchanged.

**Electrostatic force :**

Coulomb's law $\overline{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$ ; For discrete distribution of charge $\overline{F} = \overline{F}_1 + \overline{F}_2 + \ldots + \overline{F}_n$

Continuous charge distribution $\overline{F} = \frac{Q}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$

Linear: $\overline{F} = \frac{Q}{4\pi\varepsilon_0}\int_l \frac{\lambda d\ell}{r^2}\hat{r}$ ;

Surface: $\overline{F} = \frac{Q}{4\pi\varepsilon_0}\int_s \frac{\sigma ds}{r^2}\hat{r}$ ;

Volume: $\overline{F} = \frac{Q}{4\pi\varepsilon_0}\int_V \frac{\rho dV}{r^2}\hat{r}$

**Electric Field :** $\overline{F} = q\overline{E}$

Due to a point charge 'Q' : $\overline{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$ Discrete charge distribution: $\overline{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$

# UV PHYSICS ACADEMY

EMT(4)

Uniformly charged sphere (radius R) $V = \frac{q}{4\pi\varepsilon_0 R^3}\left[\frac{3R^2 - r^2}{2}\right]$; $r < R$

at the centre $V = \frac{3q}{8\pi\varepsilon_0 R}$; on the surface $V = \frac{q}{4\pi\varepsilon_0 R}$; $r = R$

charged spherical shell (radius R) $V = \frac{q}{4\pi\varepsilon_0 r}$; $r > R$, $V = \frac{q}{4\pi\varepsilon_0 R}$; $r \le R$

Circular charged disc (radius R) $V = \frac{q}{4\pi\varepsilon_0 r}$ $(r > R)$, $V = \frac{\sigma R}{2\varepsilon_0}$ (at centre), $V_{rim} = \frac{\sigma R}{\pi\varepsilon_0}$ (at rim)

Charged circular ring $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2 + r^2}}$; $r > R$, $V = \frac{q}{4\pi\varepsilon_0 R}$; $r = 0$

Infinite line of charge $V_1 - V_2 = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right)$; Due to dipole $V_{dip} = \frac{\overline{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}$

Capacitance :

Sphere (radius a) $C = 4\pi\varepsilon_0\varepsilon_r a$ ; b/w two co-axial cylinders (a < b) $C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(b/a)}$

b/w two concentric spheres of radii 'a' and 'b' (a< b) $C = 4\pi\varepsilon_0\varepsilon_r\frac{ab}{(b-a)}$

parallel plate capacitor $C = \frac{\varepsilon_0 A}{\left[d - t\left(1 - \frac{1}{k}\right)\right]}$

k= dielectric constant
t = thickness of dielectric
d= seperation b/w plates of capacitor

## UV PHYSICS ACADEMY

**ELE(5)**

**Time varying currents:**

**L – C circuit :**

$\frac{d^2q}{dt^2}+\omega^2q=\frac{E}{L}$; solution $q=q_0\sin\omega t$; $f=\frac{1}{2\pi\sqrt{LC}}$

Note: Charge on the plates of a capacitor is oscillatory in nature

**LCR circuit :**

$\frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{1}{LC}q=\frac{E}{L}$

Solution :

$q=q_0-A_1\,e^{(-k+\sqrt{k^2-\omega^2})t}-A_2e^{(-k-\sqrt{k^2-\omega^2})t}$ ; $k=\frac{R}{2L}$, $\omega=\frac{1}{\sqrt{LC}}$

case (i) : $k^2>\omega^2$ over damped (dead beat)

case (ii) : $k^2=\omega^2$ critically damped

case (iii) : $k^2<\omega^2$ under damped

Note : In under damped case the circuit is oscillatory and in other two cases circuit is non oscillatory.

The condition for oscillations in LCR ciruit $k^2<\omega^2 \Rightarrow \frac{R^2}{4L^2}<\frac{1}{LC}$

angular frequency $\beta=\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$ ; $f=\frac{\beta}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$

q = charge
C = capacitance
E = battery e.m.f
t = time
R = resistance
L = inductance
$A_1, A_2$ = amplitudes (constants)
ω = angular frequency of LC circuit
β = angular frequency of oscillations in LCR circuit

## UV PHYSICS ACADEMY

**ELE(7)**

Phase $\phi=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)$; at resonance : $X_L=X_C \Rightarrow \omega=\frac{1}{\sqrt{LC}}$; $f=\frac{1}{2\pi\sqrt{LC}}$

| Case | i) if $X_L>X_C$ | ii) if $X_L<X_C$ | iii) if $X_L=X_C$ |
|---|---|---|---|
| circuit nature is | inductive | capacitive | resistive |
| phase (φ) | i lags E by φ | i leads E by φ | i and E are in phase |

Note : The condition at which current and e.m.f are in phase is called "Resonance condition".

**Quality factor (Q):**

$Q=2\pi\frac{\text{energy stored}}{\text{energy losed per unit period}}$ (or) $Q=\frac{\text{resonant frequency}}{\text{Bandwidth}}$

$Q=\frac{X_L}{R}=\frac{\omega L}{R}$ (for L – R circuit) and $Q=\frac{X_C}{R}=\frac{1}{\omega CR}$ (for R – C circuit)

**At resonance:**

| Series LCR - circuit | Parallel LCR - circuit |
|---|---|
| $f_r=\frac{1}{2\pi\sqrt{LC}}$ | $f_r=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$ |
| z is minimum (z = R) | z is maximum (z = L/RC) |
| current is maximum | current is minimum |
| Acceptor circuit | Rejector circuit |
| Voltage magnification circuit | Current magnification circuit |

$f_r$ = resonance frequency
Z = impedence at resonance
N = no. of turns in coil
1, 2 = primary, secondary coil respectively
i = current in the coil
V = voltage
k = transformation ratio

## UV PHYSICS ACADEMY

**ELE(1)** ELECTRONICS

**Transistors :**

| Parameter | CBC | CEC | CCC |
|---|---|---|---|
| current amplification factor | $\alpha=\alpha_{dc}=\frac{I_C}{I_E}$ | $\beta=\beta_{dc}=\frac{I_C}{I_B}$ | $\gamma=\gamma_{dc}=\frac{I_E}{I_B}$ |
| | $\alpha_{ac}=\frac{\Delta I_C}{\Delta I_E}$ | $\beta_{ac}=\frac{\Delta I_C}{\Delta I_B}$ | $\gamma_{ac}=\frac{\Delta I_E}{\Delta I_B}$ |
| Range (typically) | $\alpha_{dc}=0.99$ to $0.995$ | $\beta_{dc}=20$ to $500$ | $\gamma_{dc}=\beta_{dc}+1$ |
| Total output Current | $I_C=\alpha I_E+I_{CBO}$ | $I_C=\beta I_B+I_{CEO}$ | $I_E=I_B+I_C$(general) |
| | | $I_{CEO}=\frac{I_{CBO}}{(1-\alpha)}$ | $I_E=(1+\beta)(I_B+I_{CBO})$ |

Note : 1. In a N-P-N transistor $I_B$ and $I_C$ flow into base and collector terminals respectively and $I_E$ flows out of emitter.
2. P-N-P transister $I_E$ flows into emitter and $I_B$, $I_C$ flow out of base and collector respectively.

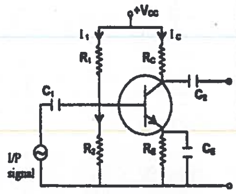
| Relation between | α, β and γ | α, β | β, γ | γ, α |
|---|---|---|---|---|
| | $\beta=\gamma\alpha$ | $\alpha=\beta/(1+\beta)$; $\beta=\alpha/(1-\alpha)$ | $\gamma=1+\beta$; $\beta=\gamma-1$ | $\gamma=1/(1-\alpha)$; $\alpha=(\gamma-1)/\gamma$ |

α, β and γ are current amplication factors in CB, CE and CC configurations respectively.
$I_E$ - emitter current, $I_B$ - base current, $I_C$ - collector current, $I_{CBO}$ - leakage current in CBC, $I_{CEO}$ - leakage current in CEC;
CBC - common base configuration; CEC - common emitter configuration, CCC - common collector config.

## UV PHYSICS ACADEMY

**ELE(3)**

**Voltage divider (or) Universal bias :**




Potential drop $V_B$ across $R_2$ is $V_B=V_{CC}\frac{R_2}{(R_2+R_1)}$

$V_B=V_{BE}+V_E$; $V_E=I_ER_E$ ; $V_{CC}=I_CR_C+V_{CE}+I_ER_E$

since $I_E=I_B+I_C$; $I_B$ is very small; $I_E\approx I_C$

Stability factor (s) : $s=\frac{1+\beta}{1-\beta\left(\frac{dI_B}{dI_C}\right)}$ ; $I_C=\beta I_B+(1+\beta)I_{CBO}$

as temperature (or) β changes $I_C$ changes

Note : 's' should be as low as possible for higher stability

**Feed back:**

for +ve feed back $A_f=\frac{A}{1-A\beta}$ for -ve feed back $A_f=\frac{A}{1+A\beta}$

The factor $|A\beta|$ is called loop gain

Brakhausen condition for sustained oscillations

i) $|A\beta|=1$ ; ii) signal must be fed in phase with the input feed back must be positive)

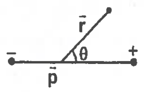
$V_{CC}$ = supply voltage
$V_{BE}$ = Base-emitter voltage
$V_C$ = collector voltage
$V_E$ = emitter voltage
A = gain without feed back
$A_f$ = gain with feed back
β = fractional value (+ve for positive feed back, –ve for negative feed back

Note: Practically to set up oscillations loop gain should be slightly greater than one ($|A\beta|>1$)

## UV PHYSICS ACADEMY

**EMT(3)**

Curl of $\bar{E}$ is $\oint\bar{E}\cdot\bar{dl}=0$




**Electric Dipole:**

$\bar{E}_{dip}(r,\theta)=\frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\hat{r}+\sin\theta\hat{\theta}\right)$ ; (θ is angle b/w $\bar{p}$ and $\bar{r}$ )

$\bar{E}$ of a (pure) dipole in the coordinate free form $\bar{E}_{dip}(r)=\frac{1}{4\pi\varepsilon_0 r^3}\left(3(\bar{p}\cdot\hat{r})\hat{r}-\bar{p}\right)$

Torque $\bar{\tau}=\bar{p}\times\bar{E}$ ; Force acting on a dipole (in non-uniform electric field) $\bar{F}=\bar{\nabla}(\bar{p}\cdot\bar{E})$

Monopole moment $Q_{total}=\int\rho dV$ ; Dipole moment $\bar{p}=\sum q_i\bar{r}_i$ (for discrete charges),

$\bar{p}=\int\rho\bar{r}dV$ (continuous charge); Quadrapole moment $Q=\int\frac{r^2}{2}(3\cos^2\theta-1)\rho dV$

**Electric potential :** $\bar{\nabla}\times\bar{E}=0$, $\bar{E}=-\bar{\nabla}V$ ; Point charge at a distance 'r' $V=\frac{q}{4\pi\varepsilon_0 r}$

Potential difference $V_a-V_b=-\int_a^b\bar{E}\cdot\bar{dl}$ ; For discrete charges $V=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$

For continuous charge distribution $V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$ ; (1D dq = λdl, 2D dq = σ ds, 3D dq = ρ dV)

Poisson's equation $\nabla^2V=-\frac{\rho}{\varepsilon_0}$ , Laplace equation $\nabla^2V=0$

## UV PHYSICS ACADEMY

**EMT(2)**

Continuous distribution $\bar{E}=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$ ; (1D dq = λdl, 2D dq = σ ds, 3D dq = ρ dV)

Flux of $\bar{E}$ through a surface 's' $\phi_E=\int_s\bar{E}\cdot\bar{da}$

Gauss's law: Integral form $\oint_s\bar{E}\cdot\bar{da}=\frac{Q_{enc}}{\varepsilon_0}$ ; Differential form : $\bar{\nabla}\cdot\bar{E}=\frac{\rho}{\varepsilon_0}$

Electric field :

uniformly charged sphere of radius 'a' $\bar{E}=\frac{q\bar{r}}{4\pi\varepsilon_0a^3}$ (r < a) , $\bar{E}=\frac{q}{4\pi\varepsilon_0r^2}\hat{r}$ (r > a)

uniformly charged disk of radius R $\bar{E}=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]\hat{x}$ ; Infinite charge sheet $\bar{E}=\frac{\sigma}{2\varepsilon_0}\hat{r}$

At mid point of two infinite charged sheets separated by a finite distance $\bar{E}=\frac{\sigma}{\varepsilon_0}\hat{r}$

Spherical shell of radius R $\bar{E}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$ ; r > R, $\bar{E}$ = zero ; r < R

Infinite spherical shell $\bar{E}=\frac{\sigma}{\varepsilon_0}\hat{r}$ ; Infinite conducting charge sheet $\bar{E}=\frac{\sigma}{\varepsilon_0}\hat{r}$

**Electrostatic boundary conditions :** 1. Tangential component is continuous $E^{\parallel}_{above}=E^{\parallel}_{below}$

2. Normal component is discontinuous $E^{\perp}_{above}-E^{\perp}_{below}=\frac{\sigma}{\varepsilon_0}\hat{n}$ ; 3. Potential is continuous $V_{above}=V_{below}$

## UV PHYSICS ACADEMY

**EMT(5)**

| Capacitors in series | Capacitors in parallel |
|---|---|
| $\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}+\ldots\frac{1}{C_n}$ | $C_p=C_1+C_2+\ldots C_n$ |
| $V_1:V_2:V_3=\frac{1}{C_1}:\frac{1}{C_2}:\frac{1}{C_3}$ | $V_1=V_2=V_3=V$ |
| 'n' identical capacitors $C_s=\frac{C}{n}$ | $C_p=nC$ |
| Multiplate capacitors: $C_{equivalent}=\frac{C}{(n-1)}$ | Multiplate capacitors: $C_{equivalent}=(n-1)C$ |

Effect of dielectric on parallel plate capacitor:

| If a dielectric is introduced b/w plates: Battery disconnected | Battery connected |
|---|---|
| $C^I=KC$ | $C^I=KC$ |
| $Q^I=Q$ | $Q^I=KQ$ |
| $V^I=V/K$ | $V^I=V$ |
| $E^I=E/K$ | $E^I=E$ |
| $U^I=U/K$ | $U^I=KU$ |

Dielectrics:(Electric fields in mattè r) $\rho_b$ = bound vol.charge density, $\sigma_b$ = bound surface charge density

Dipole moment per unit volume $\bar{p}^I=n\bar{p}=nq\bar{d}$ ; (n=no. of dipoles per unit volume) $\rho_b=-\bar{\nabla}\cdot\bar{P}$, $\sigma_b=\bar{P}\cdot\hat{n}$

$\bar{D}=\epsilon_0\bar{E}+\bar{P}=\epsilon\bar{E}$, $\chi_E=\bar{P}/\epsilon_0\bar{E}$, $\bar{P}=\alpha\bar{E}_{loc}$; $\alpha=\bar{P}/\bar{E}_{loc}$ ($\bar{P}$ = polarization of dielectric)

Gauss's law in dielectrics : Integral form $\oint\bar{D}\cdot\bar{da}=Q_{free\ enclosed}$ ; Differential form : $\bar{\nabla}\cdot\bar{D}=\rho_{free}$

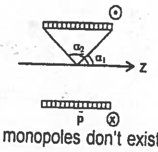
**Image charges:**

| Real charge, +q at a distance | Image point | Image charge |
|---|---|---|
| 'b' from a conducting plane | $-b$ | $-q$ |
| 'b' from a conducting sphere of radius 'a' | $a^2/b$ | $-q\ a/b$ |

## UV PHYSICS ACADEMY

**EMT(7)**

along the axis (z-axis) of a solenoid $B=\frac{\mu_0 nI}{2}(\cos\alpha_1-\cos\alpha_2)$ ;




Toroid (R – radius) $B=\frac{\mu_0 NI}{2\pi R}$ ; Ampere's law : $\oint\bar{B}\cdot\bar{dl}=\mu_0 I_{enclosed}$

Differential form $\bar{\nabla}\times\bar{B}=\mu_0\bar{J}$ ; Divergence of $\bar{B}$ : $\bar{\nabla}\cdot\bar{B}=0$ (magnetic monopoles don't exist)

**Magnetic forces :**

$\bar{F}=Q(\bar{v}\times\bar{B})$ ; Lorentz force law $\bar{F}=Q(\bar{E}+(\bar{v}\times\bar{B}))$

Current carrying conductor of length $\ell$ : $\bar{F}=I(\bar{\ell}\times\bar{B})$

Linear currents $\bar{F}_{mag}=I\int_l(d\bar{\ell}\times\bar{B})$; Surface currents $\bar{F}_{mag}=\int_s(\bar{K}\times\bar{B})da$; Volume currents $\bar{F}_{mag}=\int_v(\bar{J}\times\bar{B})dv$

K = surface current density, $\bar{K}=dI/d\ell_\perp=\sigma\bar{v}$
J = vol. current density $=dI/da_\perp=\rho\bar{v}$
$\phi$ = magnetic flux density

Force per unit length between two long parallel current carrying straight wires $\frac{dF}{d\ell}=\frac{\mu_0 I_1 I_2}{2\pi r}$

Electromagnetic Induction :
(A changing magnetic field induces an electric field)

$\varepsilon=-\frac{d\phi}{dt}$ ; Faraday's law: Integral form $\oint\bar{E}\cdot\bar{dl}=-\int\frac{\partial\bar{B}}{\partial t}\cdot\bar{da}$

Differential form $\bar{\nabla}\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}$ ; Self inductance: $L=\varepsilon/\left(-\frac{dI}{dt}\right)$ ;

$\bar{E}$ = induced electric field
$n=\frac{N}{\ell}$; $V(=\ell a)$ volume
N= total number of turns
$\varepsilon$ = induced e.m.f.

for a solenoid ; two co-axial cables - inner radius 'a' outer radius 'b' $L=\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$

## UV PHYSICS ACADEMY

**EMT(9)**

$\rho_e$- electric charge density, $J_e$- electric current density, $\rho_m$ - magnetic charge density, $J_m$ - magnetic current density

**Boundary conditions :**

| In free space | The boundary is a conducting surface (In free sapce) | In linear media (Boundary is the interface of two media) |
|---|---|---|
| $E_1^\perp-E_2^\perp=0$ | $E_1^\perp-E_2^\perp=\frac{\sigma}{\epsilon_0}$ | $D_1^\perp-D_2^\perp=\sigma_f$ or $(\epsilon_1 E_1^\perp-\epsilon_2 E_2^\perp=\sigma_f)$ |
| $E_1^\parallel-E_2^\parallel=0$ | $E_1^\parallel-E_2^\parallel=0$ | $E_1^\parallel-E_2^\parallel=0$ <br> $(D_1^\parallel-D_2^\parallel=P_1^\parallel-P_2^\parallel)$ |
| $B_1^\perp-B_2^\perp=0$ | $B_1^\perp-B_2^\perp=0$ | $B_1^\perp-B_2^\perp=0$ |
| $B_1^\parallel-B_2^\parallel=0$ | $B_1^\parallel-B_2^\parallel=\mu_0(\bar{K}\times\hat{n})$ | $H_1^\parallel-H_2^\parallel=(\bar{K}_f\times\hat{n})$ <br> $\frac{B_1^\parallel}{\mu_1}-\frac{B_2^\parallel}{\mu_2}=(\bar{K}_f\times\hat{n})$ |

$E^\perp$ = normal component of $\bar{E}$
$E^\parallel$ = tangential component of $\bar{E}$
$B^\perp$ = normal component of $\bar{B}$
$B^\parallel$ = tangential component of $\bar{B}$
$\bar{K}_f$ = free surface current density

**Poynting vector :** (Rate of flow of energy per unit area per unit time)

$\bar{S}=\frac{\bar{E}\times\bar{B}}{\mu_0}=\bar{E}\times\bar{H}$; Energy density $u=\left(\frac{1}{2}\epsilon_0 E^2+\frac{B^2}{2\mu_0}\right)$

## UV PHYSICS ACADEMY

**EMT(11)**

The solutions of wave equations are: $\bar{E}=E_0\cos(kx-\omega t+\delta)$; $\bar{B}=B_0\cos(kx-\omega t+\delta)$

Monochromatic polarized electromagnetic wave $\tilde{E}=\tilde{E}_0 e^{i(kx-\omega t)}\hat{y}$; $\tilde{B}=\frac{k}{\omega}(\hat{x}\times\tilde{E})$

The solution of plane polarized wave (polarized in $\hat{y}$ -direction

$E=E_0\cos(kx-\omega t+\delta)\hat{y}$, $B=\frac{k}{\omega}E_0\cos(kx-\omega t+\delta)\hat{z}$

**EM waves :**

| In free space | In medium |
|---|---|
| $\langle u\rangle=\frac{1}{2}\epsilon_0 E_0^2=\frac{1}{2}u_{max}$ | $\langle u\rangle=\frac{1}{2}\epsilon E_0^2$ |
| $I=\langle s\rangle=\frac{1}{2}c\epsilon_0 E_0^2$ | $I=\langle s\rangle=\frac{1}{2}v\epsilon E_0^2$ |
| $\langle P\rangle=\frac{1}{2c}\epsilon_0 E_0^2$ | $\langle P\rangle=\frac{1}{2v}\epsilon E_0^2$ |
| $v=c=\frac{1}{\sqrt{\mu_0\epsilon_0}}$ | $v=\frac{1}{\sqrt{\mu\epsilon}}=\frac{c}{\sqrt{\mu_r\epsilon_r}}$ |

<u>= average energy density
<s> = average value of poynting theorem
I = intensity of EM fields in free space
$\langle P\rangle$ = average value of momentum density
v = velocity of EM wave
$\mu_r$ = relative permeability
$\epsilon_r$ = relative permetivity
$\mu$ = permeability of medium
$\varepsilon$ = permetivity of medium
$n_1, n_2$ = refractive indeces of medium 1 and 2 respectively
$v_1, v_2$ = velocities of EM wave in medium 1 and 2 respectively.

R and T of an EM wave for normal incidence

$R=\left(\frac{v_2-v_1}{v_1+v_2}\right)^2=\left(\frac{n_1-n_2}{n_1+n_2}\right)^2$; $T=\frac{4v_1v_2}{(v_1+v_2)^2}=\frac{4n_1n_2}{(n_1+n_2)^2}$; $R+T=1$

## UV PHYSICS ACADEMY

**EMT(13)**

Skin depth $d=\frac{1}{k_I}=\frac{1}{\omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1\right]^{1/2}}$ ;

for good conductor $(\sigma>>\epsilon\omega)$ $d=\sqrt{\frac{2}{\mu\sigma\omega}}$

for poor conductor $(\sigma<<\epsilon\omega)$ $d=\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$

Wave equations: $\nabla^2\bar{E}=\mu\sigma\frac{\partial\bar{E}}{\partial t}+\mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2}$ $\quad\nabla^2\bar{B}=\mu\sigma\frac{\partial\bar{B}}{\partial t}+\mu\epsilon\frac{\partial^2\bar{B}}{\partial t^2}$

general solutions of above equations $\tilde{E}=\tilde{E}_0 e^{i(kx-\omega t)}$; $\tilde{B}=\tilde{B}_0 e^{i(kx-\omega t)}$

If the wave is polarized in y-direction : real part of $\tilde{E}$ : $\bar{E}=E_0 e^{-k_I x}\cos(k_R x-\omega t+\delta_E)\hat{y}$

real part of $\tilde{B}$ : $\bar{B}=\frac{\sqrt{k_R^2+k_I^2}}{\omega}E_0 e^{-k_I x}\cos(k_R x-\omega t+\delta_E+\varphi)\hat{z}$

Note :In conductors $\bar{E}$ and $\bar{B}$ are not in phase and $\bar{E}$ lags $\bar{B}$ by $\phi$.

Gauge transformations : $V^I=V-\frac{\partial\lambda}{\partial t}$, $A^I=A+\nabla\lambda$

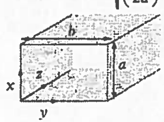
## UV PHYSICS ACADEMY

**EMT(15)**

Waveguides

Wave guide equation : $k_g^2=\frac{\omega^2}{c^2}-\frac{m^2\pi^2}{a^2}-\frac{n^2\pi^2}{b^2}$ Guide cutoff frequency $\nu_c=c\sqrt{\left(\frac{m}{2a}\right)^2+\left(\frac{n}{2b}\right)^2}$

Phase velocity above cutoff $V_p=\frac{c}{\sqrt{1-(\nu_c/\nu)^2}}$




group velocity above cutoff $V_g=c^2/\upsilon_p=c\sqrt{1-(\nu_c/\nu)^2}$

| $TE_{mn}$ mode | FIELD SOLUTIONS | $TM_{mn}$ mode | |
|---|---|---|---|
| $B_x=\frac{ik_g c^2}{\omega_c^2}\frac{\partial B_z}{\partial x}$ | $E_x=\frac{i\omega c^2}{\omega_c^2}\frac{\partial B_z}{\partial y}$ | $E_x=\frac{ik_g c^2}{\omega_c^2}\frac{\partial E_z}{\partial x}$ | $B_x=-\frac{i\omega}{\omega_c^2}\frac{\partial E_z}{\partial y}$ |
| $B_y=\frac{ik_g c^2}{\omega_c^2}\frac{\partial B_z}{\partial y}$ | $E_y=-\frac{i\omega c^2}{\omega_c^2}\frac{\partial B_z}{\partial x}$ | $E_y=\frac{ik_g c^2}{\omega_c^2}\frac{\partial E_z}{\partial y}$ | $B_y=\frac{i\omega}{\omega_c^2}\frac{\partial E_z}{\partial x}$ |
| $B_z=B_0\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$ | $E_z=0$ | $E_z=E_0\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$ | $B_z=0$ |
| Wave impedence $Z_{TE}=Z_0/\sqrt{1-(\nu_c/\nu)^2}$ | | Wave impedence | $Z_{TM}=Z_0\sqrt{1-(\nu_c/\nu)^2}$ |

$E_x=E_{0x}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-ik_g z}$ $\quad E_y=E_{0y}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-ik_g z}$ $\quad E_z=0$

$B_x=B_{0x}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-ik_g z}$ $\quad B_y=B_{0y}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-ik_g z}$ $\quad B_z=B_{0z}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-ik_g z}$

## UV PHYSICS ACADEMY

**EMT(8)**

**Scalar & vector potential :** $\nabla\times\bar{E}=0 \Rightarrow \bar{E}=-\bar{\nabla}V$ ; $\quad \bar{\nabla}\cdot\bar{B}=0; \bar{B}=\bar{\nabla}\times\bar{A}$ ; $\quad \bar{\nabla}\cdot\bar{A}=0$ ;

$\nabla^2\bar{A}=-\mu_0\bar{J}$ ; $\quad \bar{A}(r)=\frac{\mu_0}{4\pi}\int\frac{\bar{J}(r)}{r}dV$ ; $\quad \bar{A}_{dip}(r)=\frac{\mu_0}{4\pi}\frac{\bar{m}\times\hat{r}}{r^2}$ ; Mag.dipole moment $m=NI\int da=NIa$ ;

bound current density $\bar{J}_b=\bar{\nabla}\times\bar{M}$ ; Surface current density $\bar{K}_b=\bar{M}\times\hat{n}$

Ampere's law in magnetized materials $\bar{\nabla}\times\bar{H}=\bar{J}_f$ (or) $\oint\bar{H}\cdot d\bar{l}=I_{f\,encl}$ ; $H=\frac{B}{\mu_0}-M$

Maxwell's equations:

| | General form | In free space ($\rho=0$, $J=0$) | Hypothetical case (if magnetic poles exist) | In matter | In matter ($\rho_f=0$, $J_f=0$) |
|---|---|---|---|---|---|
| Maxwell I | $\bar{\nabla}\cdot\bar{E}=\frac{\rho}{\epsilon_0}$ | $\bar{\nabla}\cdot\bar{E}=0$ | $\bar{\nabla}\cdot\bar{E}=\frac{\rho_e}{\epsilon_0}$ | $\bar{\nabla}\cdot\bar{D}=\rho_f$ | $\bar{\nabla}\cdot\bar{D}=0$ |
| Maxwell II | $\bar{\nabla}\cdot\bar{B}=0$ | $\bar{\nabla}\cdot\bar{B}=0$ | $\bar{\nabla}\cdot\bar{B}=\mu_0\rho_m$ | $\bar{\nabla}\cdot\bar{B}=0$ | $\bar{\nabla}\cdot\bar{B}=0$ |
| Maxwell III | $\bar{\nabla}\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}$ | $\bar{\nabla}\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}$ | $\bar{\nabla}\times\bar{E}=-\mu_0\bar{J}_m-\frac{\partial\bar{B}}{\partial t}$ | $\bar{\nabla}\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}$ | $\bar{\nabla}\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}$ |
| Maxwell IV | $\bar{\nabla}\times\bar{B}=\mu_0\bar{J}+\mu_0\epsilon_0\frac{\partial\bar{E}}{\partial t}$ | $\bar{\nabla}\times\bar{B}=\mu_0\epsilon_0\frac{\partial\bar{E}}{\partial t}$ | $\bar{\nabla}\times\bar{B}=\mu_0\bar{J}_e+\mu_0\epsilon_0\frac{\partial\bar{E}}{\partial t}$ | $\bar{\nabla}\times\bar{H}=\bar{J}_f+\frac{\partial\bar{D}}{\partial t}$ | $\bar{\nabla}\times\bar{H}=\frac{\partial\bar{D}}{\partial t}$ |

## UV PHYSICS ACADEMY

**EMT(6)**

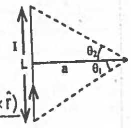
**Potential energy:**

System of point charges $U=\frac{1}{8\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_iq_j}{r_{ij}}$ ; For discrete charges $U=\frac{1}{2}\sum_i q_iV_i$ ,

Continuous distribution of charge $U=\frac{1}{2}\int V(r)dq$ ; Energy stored in a spherical shell $U=\frac{Q^2}{8\pi\varepsilon_0R}$

Self energy of a uniformly charged sphere $U=\frac{3Q^2}{20\pi\varepsilon_0R}$ ; Electric dipole $U=-\bar{p}\cdot\bar{E}$ ;

Electrostatic pressure $P_{es}=\frac{\sigma^2}{2\varepsilon_0}=\frac{1}{2}\varepsilon_0E^2$




Magnetic fields :

Biot-Savart's law $\bar{B}=\frac{\mu_0}{4\pi}\int\frac{I(\bar{dl}\times\hat{r})}{r^2}$ ; due to moving charge $\bar{B}=\frac{\mu_0}{4\pi}\frac{q(\bar{v}\times\hat{r})}{r^2}$

due to thin straight conductor of finite size $\bar{B}_{axis}=\frac{\mu_0I}{4\pi a}(\sin\theta_1+\sin\theta_2)\hat{z}$

circular current carrying loop $\bar{B}_x=\frac{\mu_0Ia^2}{2(x^2+a^2)^{3/2}}\hat{x}$; $x>a$ ; arc of an angle $\theta$ : $B=\frac{\mu_0I}{4\pi a}\theta$

long cylindrical tube (radius R) $\bar{B}=\frac{\mu_0I}{2\pi r}\hat{\phi}$; $r>R$ , $\bar{B}=$ zero; $r<R$

Solid cylinder (radius R) $\bar{B}=\frac{\mu_0I}{2\pi r}\hat{\phi}$; $r>R$ ; Inside solid cylinder $\bar{B}=\frac{\mu_0Ir}{2\pi R^2}\hat{\phi}$; $r<R$

## UV PHYSICS ACADEMY

**EMT(12)**

Oblique Incidence $\theta<90^0$

| | polarization is in plane of incidence | perpendicular to the plane of incidence |
|---|---|---|
| Fresnel's equations | $\tilde{E}_{0T}=\left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$ <br> $\tilde{E}_{0R}=\left(\frac{\beta-\alpha}{\beta+\alpha}\right)\tilde{E}_{0I}$ | $\tilde{E}_{0T}=\left(\frac{2}{(1+\alpha\beta)}\right)\tilde{E}_{0I}$ <br> $\tilde{E}_{0R}=\left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)\tilde{E}_{0I}$ |
| Reflection coefficient (R) | $\frac{I_R}{I_I}=\left(\frac{\tilde{E}_{0R}}{\tilde{E}_{0I}}\right)^2=\left(\frac{\beta-\alpha}{\beta+\alpha}\right)^2$ | $\frac{I_R}{I_I}=\left(\frac{\tilde{E}_{0R}}{\tilde{E}_{0I}}\right)^2=\left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2$ |
| Transmission coefficient (T) | $\frac{I_T}{I_I}=\left(\frac{\tilde{E}_{0T}}{\tilde{E}_{0I}}\right)^2=\frac{4\alpha\beta}{(\alpha+\beta)^2}$ | $\frac{I_T}{I_I}=\left(\frac{\tilde{E}_{0T}}{\tilde{E}_{0I}}\right)^2=\frac{4}{(1+\alpha\beta)^2}$ |

EM waves in a conductor:

$\rho_f(t)=\rho_f(0)e^{-\frac{\sigma}{\epsilon}t}$ ; $k=k_R+ik_I$ ; k - total dielectric constant

$k_R=\omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}+1\right]^{1/2}$ , $k_I=\omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1\right]^{1/2}$

$\alpha=\frac{\mu_1v_2}{\mu_2v_1}$, $\beta=\frac{\cos\theta_T}{\cos\theta_I}$

$\rho_f(t)$ = free vol. current density of a conductor in a trnasient state at any instant of time 't'.

## UV PHYSICS ACADEMY

**EMT(10)**

equation of continuity (conservation of charge) $\bar{\nabla}\cdot\bar{J}=-\frac{\partial\rho}{\partial t}$

conservation of energy in EM fields (poynting theorem) $\frac{\partial}{\partial t}(u_{mech}+u_{em})=-\bar{\nabla}\cdot\bar{S}$

conservation of force in EM fields $\bar{F}=\oint_s\bar{T}\cdot d\bar{a}-\mu_0\epsilon_0\frac{d}{dt}\int\bar{S}dV$

Note : If the fields are static then second term disappears

conservation of momentum in EM fields $\bar{\nabla}\cdot(-\bar{T})=-\frac{d}{dt}(\bar{P}_{mech}+\bar{P}_{em})$

Reflection and transmission of a wave in a string

$T=\left|\frac{\tilde{A}_T}{\tilde{A}_I}\right|^2=\left(\frac{2k_1}{k_1+k_2}\right)^2=\left(\frac{2v_2}{v_1+v_2}\right)^2$

$R=\left|\frac{\tilde{A}_R}{\tilde{A}_I}\right|^2=\left(\frac{k_1-k_2}{k_1+k_2}\right)^2=\left(\frac{v_2-v_1}{v_1+v_2}\right)^2$

Wave equation in 3D

$\nabla^2\bar{E}=\frac{1}{c^2}\frac{\partial^2\bar{E}}{\partial t^2}$; $\nabla^2\bar{B}=\frac{1}{c^2}\frac{\partial^2\bar{B}}{\partial t^2}$; $c^2=\frac{1}{\mu_0\epsilon_0}$

$\bar{S}$ = poynting vector
$u_{mech}$ = mechanical energy density
$u_{em}$ = electromagnetic energy density
$\bar{T}$ maxwell'sstresstensor
$P_{mech}$ = mechanical momentum density
$P_{em}$ = electromagnetic momentum density
$k_1$, $k_2$ = propagation constants in medium 1, 2 respectively.
$v_1$, $v_2$ = velocities of EM waves in medium 1, 2 respectively.

## UV PHYSICS ACADEMY

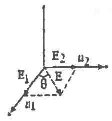
**EMT(16)**

Polarisation of Elelctromagnetic waves

Direction of Electric field Vector E = State of Polarisaton
Direction of Propagation = Plane of vibration

$E_0=\sqrt{E_{01}^2+E_{02}^2}$

$\theta=\tan^{-1}=\frac{E_{02}}{E_{01}}$




<u>Plane polarisation</u>

$E_1=\hat{n}_1E_1^0e^{i(\omega t-kz)}$

$E(r,t)=(\hat{n}_1E_{01}+\hat{n}_2E_{02})e^{i(\omega t-kz)}$ $\quad E_2=\hat{n}_2E_2^0e^{i(\omega t-kz)}$

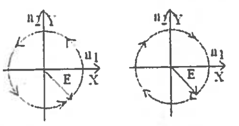
Phase difference between $E_{01}$ and $E_{02}$ is $\phi=0$ (in phase)

<u>Circualry polarised</u>

$E_x(r,t)=E_0\cos(\omega t-kz)$

$E(r,t)=E_0(\hat{n}_1\pm i\hat{n}_2)e^{i(\omega t-kz)}$ $\quad E_y(r,t)=\mp E_0\sin(\omega t-kz)$

Phase difference between $E_{01}$ and $E_{02}$ is $\phi=\pi/2$
and $E_{01}=E_{02}=E_0$ $\quad E_0^2=E_x^2+E_y^2$




<u>Left handed</u> $\hat{n}_1+i\hat{n}_2$ $\quad$ <u>Right handed</u> $\hat{n}_1-i\hat{n}_2$

<u>Eliptically polarised</u>

Phase difference between $E_{01}$ and $E_{02}$ is $\phi=\pi/2$
and $E_{01}\neq E_{02}$ $\quad E_x(r,t)=E_0\cos(\omega t-kz)$ $\quad E_y(r,t)=\mp E_0\sin(\omega t-kz)$ $\quad \frac{E_x^2}{E_{01}^2}+\frac{E_y^2}{E_{01}^2}=1$

| Radiation due to Rrelativistic charges $v>>c$: | Radiation due to Non relativistic charges $v<<c$: |
|---|---|
| Lienard relation $\gamma=1/\sqrt{1-\beta^2}$ | Poynting theorem $P=\frac{\hat{n}}{4\pi\varepsilon_0}\left(\frac{e^2v^2}{4\pi c^3r^2}\right)\sin^2\theta$ |
| Power radiated $P_t=\frac{1}{4\pi\varepsilon_0}\left(\frac{2e^2\gamma^6}{c}\right)\left[\dot{\beta}^2-(\beta\times\dot{\beta})^2\right]$ | Power radiated $P_t=\frac{1}{4\pi\varepsilon_0}\left(\frac{2e^2v^2}{3c^3}\right)$ Larmor Relation |

## UV PHYSICS ACADEMY

**EMT(14)**

Gauge conditions

Coulomb guage condition $\nabla\cdot A=0$

Lorentz gauge condition $\bar{\nabla}\cdot\bar{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0$

$\Box^2$ = D'Alembertain opr
J = current density
$\rho$ = volumecharge density
$\Box^2=\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

**Potential fields equations:**

$\Box^2\bar{A}=\mu_0\bar{J}$ $\quad \Box^2\phi=-\frac{\rho}{\epsilon_0}$ $\quad \bar{B}=\bar{\nabla}\times\bar{A}$; $\bar{E}=-\bar{\nabla}\phi-\frac{\partial\bar{A}}{\partial t}$

$\phi(r,t)=\frac{1}{4\pi\epsilon_0}\int_v\frac{\rho\left(r,t-\frac{r}{c}\right)}{r}dV$ $\quad A(r,t)=\frac{\mu_0}{4\pi}\int_v\frac{\bar{J}\left(r,t-\frac{r}{c}\right)}{r}dV$

Lienard- Wichert potentials: $\bar{A}=\frac{\mu_0}{4\pi}\frac{q\bar{v}}{\left(r-\frac{\bar{v}\cdot\bar{r}}{c}\right)}$ $\quad \phi=\frac{1}{4\pi\epsilon_0}\frac{q}{\left(r-\frac{\bar{v}\cdot\bar{r}}{c}\right)}$

Maxwell's Equations interms of $\phi$ and A:

i) $\nabla^2\phi+\frac{\partial}{\partial t}(\bar{\nabla}.\bar{A})=\frac{-\rho}{\epsilon_0}$ $\quad$ iii) $\bar{\nabla}\times(-\bar{\nabla}\phi-\frac{\partial\bar{A}}{\partial t})=-\frac{\partial}{\partial t}(\bar{\nabla}\times\bar{A})$

ii) $\bar{\nabla}.(\bar{\nabla}\times\bar{A})=0$ $\quad$ iv) $\nabla(\nabla.A)+\mu_0\epsilon_0\frac{\partial\phi}{\partial t})-(\nabla^2A-\mu_0\epsilon_0\frac{\partial^2A}{\partial t^2})=\mu_0\bar{J}$

Generating function : $\dfrac{e^{-xz/(1-z)}}{(1-z)} = \sum_{n=0}^{\infty} z^n L_n(x)$ ; Rodrigue's formual : $L_n = \dfrac{e^x}{n!}\dfrac{d^n}{dx^n}(x^n e^{-x})$

Recurrence relation : $(n+1)L_{n+1} = (2n-x+1)L_n - nL_{n-1}$ , $xL_n^{l} = n[L_n - L_{n-1}]$, $L_n^{l} = -\sum_{r=0}^{n-1} L_r$

**Associated forms of equations :**

Legenders equation : $\dfrac{d}{dx}\left[(1-x^2)\dfrac{dP_\ell^m}{dx}\right] + \left[\ell(\ell+1) - \dfrac{m^2}{1-x^2}\right]P_\ell^m = 0$

Polynomial : $P_\ell^m = (1-x^2)^{m/2}\dfrac{d^m}{dx^m}P_\ell \quad 0 \le m \le \ell$

$P_\ell^{-m} = (-1)^m \dfrac{(\ell-m)!}{(\ell+m)!}P_\ell^m$

Recurrence relation :

$P_m^m = (-1)^m (2m-1)!(1-x^2)^{m/2}$

$(\ell-m+1)P_{\ell+1}^m = (2\ell+1)x P_\ell^m - (\ell+m)P_{\ell-1}^m$

Orthogonality : $\int_{-1}^{1} P_p^m(x)P_q^m(x)dx = \dfrac{1}{2q+1}\left[\dfrac{(q+m)!}{(q-m)!}\right]\delta_{p,q}$

Generating function : $\dfrac{2m!(1-x^2)^{m/2}}{2^m m!(1-2xz+z^2)^{m+1/2}} = \sum_{s=0}^{\infty} P_{s+m}^m z^s$

$P_1(1-P_2) \rightarrow$ first event happen & second fails ; $(1-P_1)(1-P_2) \rightarrow$ both event fails

$1-(1-P_1)(1-P_2) \rightarrow$ at least one of the event happen

**Special Functions :** Legendre's equation : $(1-x^2)\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + n(n+1)y = 0$

Polynomial : $P_n(x) = \sum_{r=0}^{\infty} \dfrac{(-1)^r(2n-2r)!}{2^n r!(n-r)!(n-2r)!}x^{n-2r}$, $\left(\dfrac{n}{2}\right)$ — $\dfrac{n}{2}$ if 'n' is even; $\dfrac{n-1}{2}$ if 'n' is odd

Rodrigue formula : $P_n(x) = \dfrac{1}{2^n n!}\dfrac{d^n}{dx^n}(x^2-1)^n$ ; Generating function : $(1-2xz+z^2)^{-1/2} = \sum_{n=0}^{\infty} z^n P_n(x)$

Orthogonality : $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$ if $m \ne n$

$= \dfrac{2}{2n+1}$ if $m = n$

Recurrence relation :

i) $nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-1}$

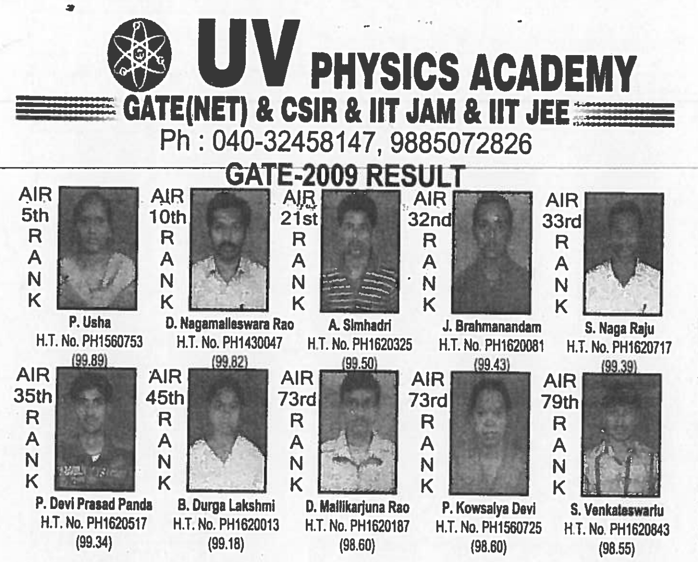
ii) $nP_n = xP_n^{l} - P_{n-1}^{l}$

iii) $(2n+1)P_n = P_{n+1}^{l} - P_{n-1}^{l}$

iv) $(n+1)P_n = P_{n+1}^{l} - xP_n^{l}$

Bessel's equation : $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2-n^2)y = 0$

Neumann's function : $N_n = \dfrac{j_n \cos n\pi - J_{-n}}{\sin n\pi}$






**MMP(18)**

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \rightarrow$ rank $= r$ $\quad K = \begin{bmatrix} a_{11} & a_{12} & a_{13} & k_1 \\ a_{21} & a_{22} & a_{23} & k_2 \\ a_{31} & a_{32} & a_{33} & k_3 \end{bmatrix} \rightarrow$ rank $= r^l$

n = number of un knowns, m = number of equations.

1) $r \ne r^l$, equations are consistent, no solution.
2) $r = r^l = n$, equations are consistent and unique solution.
3) $r = r^l < n$, equations are consistent and infinite number of solutions.

**For homogeneous equations :** $a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n = 0, \ldots, a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = 0$

r = n, only trivial zero soution ; r < n, (n – r) linearly independent solution
m < n, infinite solutions; m = n, non-trivial solution

**Properties of Dirac Delta function:**

The delta function is an even function : $\delta(-x) = \delta(x)$ ; $x\delta(x) = 0$ ; $x\delta(x-x_0) = x_0\delta(x-x_0)$

$f(x)\delta(x-x_0) = f(x_0)\delta(x-x_0)$ ; $\delta(ax) = \dfrac{1}{a}\delta(x),\ a > 0$ ; $\int \delta(x-b)\delta(a-x)dx = \delta(a-b)$

$\delta(x^2-a^2) = \dfrac{1}{2|a|}[\delta(x-a) + \delta(x+a)]$ ; $\delta(x) = 0$ for $x \ne 0$ and $\delta(x) = \infty$ for $x = 0$ such that $\int_{-\infty}^{\infty} \delta(x)dx = 1$

By making change of origin :

$\delta(x-x_0) = 0$ for $x \ne x_0$ and $\delta(x-x_0) = \infty$ for $x = 0$ , such that $\int_{-\infty}^{+\infty} \delta(x-x_0)dx = 1$

## UV PHYSICS ACADEMY

'(13)

### Probability

ability of an event = $\dfrac{\text{number of cases in which event occurs}}{\text{total number of cases}}$

ent occur 'm' times out of 'n' trials, probability of event $p = \underset{n\to\infty}{Lt}\dfrac{m}{n}$,

$\leq 1$ → for impossible event $p = 0$; for sure event $p = 1$

mial distribution :

$= r) = \dfrac{n!}{(n-r)!\,r!}P^r(1-P)^{n-r}$ , $r = 0, 1, 2 \dots n$

son distribution :

$= n) = \dfrac{e^{-\lambda}\lambda^n}{n!}$ , $n = 0, 1, 2\dots$

nal distribution :

$= \dfrac{1}{\sigma\sqrt{2\pi}}\exp\left[-\dfrac{1}{2}\left[\dfrac{x-\mu}{\sigma}\right]^2\right]$, $-\infty < x < \infty$

For mutually exclusive event :

$P(A_1 + A_2 + \dots + A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$

$P(A_1 A_2 \dots A_r) = P(A_1)\,P(A_2)\dots P(A_r)$

If A, B are not mutually exclusive

$P(A+B) = P(A) + P(B) - P(AB)$

If A & B are exclusive

$P(AB) = 0 \Rightarrow P(A+B) = P(A) + P(B)$

$P(AB) = P(A)P\left(\dfrac{B}{A}\right)$

= probability of event A to happen ; $P(\bar{A})$ = probability of event A not to happen

+ B) = probability of occurence of at least A, B; P(AB) = probability of occurence of at least

B; $P\left(\dfrac{B}{A}\right)$ = conditional probability for event B to happen when event A has already happend

## UV PHYSICS ACADEMY

MMP(15)

### Bessel's pôlynomial

Polynomial : $J_n = \sum_{r=0}^{\infty}(-1)^r \dfrac{1}{r!\,\Gamma(n+r+1)}\left(\dfrac{x}{2}\right)^{n+2r}$ , $J_{-n} = (-1)^n J_n$ ; Generating function : $e^{\frac{x}{2}\left(z-\frac{1}{z}\right)} = \sum_{-\infty}^{\infty} z^n J_n(x)$

Orthogonality :

$$\int_0^a xJ_n(\lambda_i x)J_n(\lambda_j x)\,dx = \begin{cases} 0 & \text{if } i \neq j \\ \dfrac{a^2}{2}J_{n+1}^2(\lambda_i x) & \text{if } i = j \end{cases}$$

$\lambda_i$, $\lambda_j$ are roots of $J_n(\lambda x) = 0$

Recurrence relation :

$xJ_n' = nJ_n - xJ_{n+1}$ ; $\dfrac{d}{dx}(x^n J_n) = x^n J_{n-1}$

$\dfrac{d}{dx}(x^{-n}J_n) = -x^{-n}J_{n+1}$ ; $J_n' = \dfrac{n}{x}J_n - J_{n+1}$

Bessel Integral :

$J_n = \dfrac{1}{\pi}\int_0^{\pi}\cos(n\phi - x\sin\phi)\,d\phi$ ;

Jacobi series :

$\cos x = J_0 - 2J_2 + 2J_4 - \dots$, $\sin x = 2J_1 - 2J_3 + 2J_5 - \dots$

Hermite equation : $\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2ny = 0$ ; Generating function $e^{2xt-t^2} = \sum_{n=0}^{\infty}\dfrac{H_n}{n!}t^n$

Polynomial : $H_n = \sum_{r=0}^{n/2}(-1)^r \dfrac{n!}{r!(n-2r)!}(2x)^{n-2r}$, $\left(\dfrac{n}{2}\right) = \dfrac{n}{2} \to$ even , $\left(\dfrac{n}{2}\right) = \dfrac{n-1}{2} \to$ odd

Rodrigue's formula $H_n = (-1)^n e^{x^2}\dfrac{d^n}{dx^n}e^{-x^2}$

Orthogonality : $\int_{-\infty}^{\infty}e^{-x^2}H_m H_n\,dx = 0,\ m \neq n$

$= 2^n n!\sqrt{\pi},\ m = n$

Recurrence relation :

$H_n' = 2nH_{n-1}$, $H_{n+1} = 2[xH_n - nH_{n-1}]$

$H_n'' = 2xH_n' - 2nH_n$, $H_n' = 2xH_n - H_{n+1}$

## UV PHYSICS ACADEMY

MP(17)

### Associated forms of equations :

guerre equation : $x\dfrac{d^2y}{dx^2} + (k+1-x)\dfrac{dy}{dx} + ny = 0$ ; Polynomial : $L_n^k = \sum_{r=0}^{\infty}(-1)^r \dfrac{(n+k)!}{(n-r)!(k+r)!\,r!}x^r$

enerating function : $\dfrac{e^{-xz/(1-z)}}{(1-z)^{k+1}} = \sum_{n=0}^{\infty} z^n L_n^k(x)$ ; Rodrigue's formula : $L_n^k = \dfrac{e^x x^{-k}}{n!}\dfrac{d^n}{dx^n}(x^{n+k}e^{-x})$

thogonal property : $\int_0^{\infty} e^{-x}x^k L_n^k L_m^k\,dx = \dfrac{(n+k)!}{n!}\delta_{m,n}$

Recurrence relation :

$L_n^k = L_{n-1}^k + L_n^{k-1}$

$(n+1)L_{n+1}^k = (2n+k+1-x)\,L_n^k - (n+k)L_{n-1}^k$

$xL_n' = nL_n^k - (n+k)L_{n+1}^k$

### atrices: Properties

anspose : $(AB)^T = B^T A^T$ ; Conjugate : $(AB)^* = B^* A^*$ ;

onjugate transpose: $(AB)^+ = B^+ A^+$

Symmetric matrix: $A^T = A$

nti symmetric matrix : $A^T = -A$;

Hermitian: $A^+ = A$ ; Unitary matrix : $AA^+ = I$

kew Hermitian : $A^+ = -A$ ; Inverse : $A^{-1} = \dfrac{\text{adj}A}{|A|}$ ; Orthogonal matrix : $AA^T = I$

**ank of a matrix :**

inear equations are : $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = k_1$, $a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = k_2$,

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = k_3$ .

## UV PHYSICS ACADEMY

QM(13)

### Stark Effect: For ground state of H.Atom there is no stark effect

For first excited state( 4-fold) $E = 0, 0, \pm 3eEa_0$

1. One state parallel to external field
2. One state anti parallel to external field
3. Two state with zero component along field.

$\psi_+ = \dfrac{1}{\sqrt{2}}(\psi_{200} - \psi_{210})$

$\psi_- = \dfrac{1}{\sqrt{2}}(\psi_{200} + \psi_{210})$

In stark effect nth state split into (2n+1) levels

Time dependent perturbation theory : $\hat{H}(t) = \hat{H}_0 + \hat{V}(t)$

Transition probability $P_{if}(t) = \left|\dfrac{-i}{\hbar}\int_0^t <\psi_f|\hat{V}(t')|\psi_i> e^{i\omega_{fi}t'}dt'\right|^2$

Transition probability for constant perturbation $(P_{if}(t))$ $P_{if}(t) = \dfrac{4|<\psi_f|\hat{V}(t')|\psi_i>|^2}{\hbar^2\omega_{fi}^2}\sin^2\left(\dfrac{\omega_{fi}t}{2}\right)$

Transition rate $\Gamma_{if} = \dfrac{P_{if}(t)}{t} = \dfrac{2\pi}{\hbar}|<\psi_f|\hat{V}|\psi_i>|^2\,\delta(E_f - E_i)$

Fermi Golden Rule

$W_{if} = \dfrac{2\pi}{\hbar}|<\psi_f|\hat{V}|\psi_i>|^2\,\rho(E_i)$

Variational principle:

1. Selection of a trial function suitable for the given physical system $|\psi_0\rangle = |\psi_0(\alpha_1, \alpha_2, \dots)\rangle$
2. Calculate the energy of the system
   $E_0(\alpha_1, \alpha_2, \dots) = \dfrac{\langle\psi_0|H|\psi_0\rangle}{\langle\psi_0|\psi_0\rangle}$
3. Minimise $E_0$ i.e $\dfrac{\partial E_0(\alpha_1, \alpha_2, \dots)}{\partial\alpha_i} = 0 \Rightarrow$ obtain the values of $\alpha_i$
4. Substitue $\alpha_1, \alpha_2, \dots$ in $E_0$ & $\psi_0$

Trial fucntions

L.H.O Ground state: $\psi_0 = Ae^{-\alpha x^2}$

even parity,nonodes vanish at $x = \pm\infty$

First excited state: $\psi_1 = Bxe^{-\alpha x^2}$

odd parity, one node vanish at $x = \pm\infty$

Second excited state: $C(Bx^2 - 1)e^{-\alpha x^2}$

H-Atom (ground state): $\psi_0 = Ae^{-r/a}$

## UV PHYSICS ACADEMY

**MMP(4)**

### COMPLEX ANALYSIS

Cartesian form : $z = x + iy$, polar form : $z = re^{i\theta}$ Complex conjugate : $\bar{z} = x - iy = re^{-i\theta}$

$n^{th}$ root of unity : $z = (k)^{1/n} = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$ $n = 0, 1, 2, \ldots n-1$

Argument : $\theta = \arctan\frac{y}{x}$, $|z_1 + z_2| \le |z_1| + |z_2|$, $|z_1 - z_2| \ge |z_1| - |z_2|$

Logarithmic function : $\log z = \log(x+iy) + 2in\pi$ continous function $\underset{z\to z_0}{Lt} f(z) = f(z_0)$

Differentiability : $f^{l}(z_0) = \underset{z\to z_0}{Lt}\frac{f(z) - f(z_0)}{z - z_0}$

Cauchy Riemann equation : $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ Polar form : $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$

Harmonic function : $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ Cauchy's integral theorem : $\oint f(z)dz = 0$

Cauchy's integral formula : $f(z_0) = \frac{1}{2\pi i}\oint\frac{f(z)dz}{z - z_0}$

Taylor series : $f(z) = f(z_0) + \frac{f^{l}(z_0)}{1!}(z - z_0) + \frac{f^{ll}(z_0)}{2!}(z - z_0)^2 + \ldots + \frac{f^{n}(z_0)}{n!}(z - z_0)^n + \ldots$

Maclaurin series : $f(z) = f(0) + \sum_{n=1}^{\infty}\frac{f^n(0)}{n!}z^n$

## UV PHYSICS ACADEMY

**MMP(2)**

Gradient $\nabla f = \frac{\hat{q}_1}{h_1}\frac{\partial f}{\partial q_1} + \frac{\hat{q}_2}{h_2}\frac{\partial f}{\partial q_2} + \frac{q_3}{h_3}\frac{\partial f}{\partial q_3}$

Cartesian coordinate sysetm
Volume element : dxdydz
Metric element : $(h_1, h_2, h_3) = (1,1,1)$

| Rectangular (x,y,z) | Cylindrical (ρ,ϕ,z) | Spherical (r,θ,ϕ) |
|---|---|---|
| $\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$ | $\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\varphi} + \frac{\partial f}{\partial z}\hat{z}$ | $\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$ |

Divergence $\nabla . A = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}(A_1h_2h_3) + \frac{\partial}{\partial q_2}(A_2h_3h_1) + \frac{\partial}{\partial q_3}(A_3h_1h_2)\right]$

Cylindrical cordinate system
Volume element: ρdρdϕdz
Metric element$(h_1, h_2, h_3) = (1, \rho, 1)$

| Rectangular (x,y,z) | Cylindrical (ρ,ϕ,z) | Spherical (r,θ,ϕ) |
|---|---|---|
| $\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ | $\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$ | $\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$ |

Curl $\nabla \times A = \frac{1}{h_1h_2h_3}\begin{vmatrix}\hat{q}_1h_1 & \hat{q}_2h_2 & \hat{q}_3h_3 \\ \partial/\partial q_1 & \partial/\partial q_2 & \partial/\partial q_3 \\ h_1A_1 & h_2A_2 & h_3A_3\end{vmatrix}$

Spherical coordinate system
Vol element $r^2\sin\theta dr d\theta d\phi$
Metric element:$(h_1, h_2, h_3) = (1, r, r\sin\theta)$

| Rectangular (x,y,z) | Cylindrical (ρ,ϕ,z) | Spherical (r,θ,ϕ) |
|---|---|---|
| $\begin{vmatrix}\hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial\phi & \partial/\partial z \\ A_x & A_y & A_z\end{vmatrix}$ | $\begin{vmatrix}\hat{\rho}/\rho & \hat{\phi} & \hat{z}/\rho \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z\end{vmatrix}$ | $\begin{vmatrix}\hat{r}/r^2\sin\theta & \hat{\theta}/(r\sin\theta) & \hat{\phi}/r \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & rA_\phi\sin\theta\end{vmatrix}$ |

## UV PHYSICS ACADEMY

**MMP(8)**

| Laplace transform | Inverse laplace transform |
|---|---|
| $F(s) = L[F(t)] = \int_0^\infty e^{-st}F(t)dt$ | $F(t) = L^{-1}F(s) = \int_0^\infty e^{st}F(s)ds$ |
| $L[e^{at}F(t)] = F(s-a)$ | $L^{-1}[F(s-a)] = e^{at}F(t)$ |
| $L[F(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$ | $L^{-1}[F(as)] = \frac{1}{a}F\left(\frac{t}{a}\right)$ |
| $L[F^n(t)] = s^nF(s) - s^{n-1}f(0)\ldots - f^{n-1}(0)$ | $L^{-1}[F^n(s)] = (-1)^n t^n L^{-1}[F(s)]$ |
| Convolution: $F(s)G(s) = L[F(t)*G(t)]$ | $L^{-1}[F(s)G(s)] = \int_0^t F(x)G(t-x)dx$ |

<u>Laplace transform</u>

$L(1) = \frac{1}{s}$; $L(t^n) = \frac{n!}{s^{n+1}}$

$L(e^{at}) = \frac{1}{s-a}$; $L(\sin at) = \frac{a}{s^2+a^2}$

$L(\cos at) = \frac{s}{s^2+a^2}$

$L(e^{at}\sin bt) = \frac{b}{(s-a)^2+b^2}$

$L(e^{at}\cos bt) = \frac{s-a}{(s-a)^2+b^2}$

$L(e^{at}\sin hbt) = \frac{b}{(s-a)^2-b^2}$

$L(e^{at}\cos hbt) = \frac{s-a}{(s-a)^2-b^2}$

<u>Inverse laplace transform</u>

$L^{-1}\left(\frac{1}{s}\right) = 1$; $L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{(n-1)!}$ $n = 1, 2, 3\ldots$; $L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$

$L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{1}{a}\sin at$; $L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$; $L^{-1}\left(\frac{1}{(s-a)^2+b^2}\right) = \frac{1}{b}e^{at}\sin bt$;

$L^{-1}\left(\frac{s-a}{(s-a)^2+b^2}\right) = e^{at}\cos bt$ $L^{-1}\left(\frac{1}{(s^2+a^2)^2}\right) = \frac{1}{2a^3}(\sin at - at\cos at)$

## UV PHYSICS ACADEMY

**MMP(6)**

Lineardifferential equation: $\frac{dy}{dx} + P(x)y = Q(x)$ ; $y(I.F) = \int Q(x)(I.F)dx + c$; $IF = e^{\int P(x)dx}$

Bernoulis equation : $\frac{dy}{dx} + P(x)y = Q(x)y^n$ sol : $\frac{dt}{dx} + (1-n)P(x)t = Q(x)(1-n)$ $I.F = e^{\int(1-n)P(x)dx}$

| Roots of equation | Complimentary function |
|---|---|
| 1) $m_1, m_2, \ldots$ (real & different roots) | $c_1e^{m_1x} + c_2e^{m_2x} + \ldots$ |
| 2) $m_1, m_1, m_3\ldots$ (two real & equal roots) | $(c_1 + c_2x)e^{m_1x} + c_3e^{m_3x} + \ldots$ |
| 3) $\alpha + i\beta, \alpha - i\beta, m_3\ldots$ (a pair of imaginary roots) | $e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x) + c_3e^{m_3x} + \ldots$ |
| 4) $\alpha \pm i\beta, \alpha \pm i\beta, m_5\ldots$ (two pairs of imaginary roots) | $e^{\alpha x}[(c_1 + c_2x)\cos\beta x + (c_3 + c_4x)\sin\beta x] + c_5e^{m_5x}$ |

Differential equation in operator form : $y_p = \frac{1}{P(D)}Q(x)$ ; $P(D) = D^n + a_1D^{n-1} + \ldots + a_n$

i) $Q(x) = e^{ax}$, $y_p = \frac{1}{P(D)}e^{ax}$ $\begin{cases}\text{If } P(a) \ne 0 \Rightarrow y_p = \frac{1}{P(a)}e^{ax} \\ \text{If } P(a) = 0 \text{ but } P^{l}(a) \ne 0 \Rightarrow y_p = \frac{x}{P^{l}(a)}e^{ax}\end{cases}$

$\frac{1}{D-\alpha}Q = e^{\alpha x}\int Q(x)e^{-\alpha x}dx$

ii) $Q(x) = \text{const}(k)$, $y_p = \frac{1}{P(D)}k = \frac{k}{a_n}$

iii) If $Q(x) = \sin ax$ or $\cos ax$

If $\varphi(-a^2) \ne 0 \Rightarrow y_P = \frac{1}{\varphi(-a^2)}Q(x)$

If $\varphi(-a^2) = 0$, $\varphi^{l}(-a^2) \ne 0$, $y_P = \frac{1}{\varphi^{l}(-a^2)}Q(x)$

iv) If $Q(x) = e^{ax}X$, $y_p = \frac{1}{P(D)}e^{ax}X = e^{ax}\frac{1}{P(D+a)}X$

v) $Q(x) = xV \Rightarrow y_p = \frac{1}{P(D)}xV = \left[x - \frac{P^{l}(D)}{P(D)}\right]\frac{1}{P(D)}V$

vi) $\frac{1}{D^2+a^2}\sin ax = \frac{-x}{2a}\cos ax$,

vi) $\frac{1}{D^2+a^2}\cos ax = \frac{x}{2a}\sin ax$,

vii) $\frac{1}{(D-a)^k}e^{ax} = \frac{x^k}{k!}e^{ax}$, $f(a) = 0$

## UV PHYSICS ACADEMY

**MMP(12)**

Harmonic series : $\sum_{n=1}^{\infty}\frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \ldots\infty$ converges if $p > 1$ diverges if $p \le 1$

Logarithmic series : $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^n\frac{x^n}{n} + \ldots\infty$ is convergent for $-1 < x \le 1$

Power series : $a_0 + a_1x + a_2x^2 + \ldots a_nx^n + \ldots$ a's are independent of 'x'

Exponential series : $e^x = 1 + x + \frac{x^2}{2!} + \ldots$ converges for all 'x'

Binomial series : $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots\infty$

Converges for $|x| < 1$

Logarithmic test :

$\underset{n\to\infty}{Lt}\left[n\log\frac{u_n}{u_{n+1}}\right] = k$
- converges if $k > 1$
- diverges if $k < 1$
- Fails if $k = 1$

Cauchy's root test :

$\underset{n\to\infty}{Lt}(u_n)^{1/n} = \lambda$
- converges if $\lambda < 1$
- diverges if $\lambda > 1$

D'Alembert's ratio test :

$\underset{n\to\infty}{Lt}\frac{u_{n+1}}{u_n} = \lambda$
- $\sum_n u_n$ converges if $\lambda < 1$
- $\sum_n u_n$ diverges if $\lambda > 1$

Lebinitz's test : An alternating series converge if

i) $u_n < u_{n-1}$ ii) $\underset{n\to\infty}{Lt}u_n = 0$ iii) if $\underset{n\to\infty}{Lt}u_n \ne 0$ series diverges

## UV PHYSICS ACADEMY

**MMP(10)**

f(x)
- Periodic, single valued finite
- Finite number of discontinuties in any period
- Must have finite number of maxima and minima

Fourier integral : $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}f(t)\left[\int_{-\infty}^{\infty}\cos u(x-t)du\right]dt$

For even function : $f(x) = \frac{2}{\pi}\int_0^\infty f(t)dt\int_0^\infty \cos ut\cos ux\, du$; For odd function : $f(x) = \frac{2}{\pi}\int_0^\infty f(t)dt\int_0^\infty \sin ut\sin ux\, du$

In complex form : $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{iux}du\int_{-\infty}^{\infty}f(t)e^{-iut}du$

Fourier Transform :

| Fourier transform | Inverse Fourier transform |
|---|---|
| $F[f(x)] = f(\bar{s}) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{isx}dx$ | $F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-isx}dx = f(-\bar{s})$ |
| Cosine $F_c[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\,dx = f_c(\bar{s})$ | Inverse cosine $f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_c(\bar{s})\cos sx\,ds$ |
| Sine $F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx = f_s(\bar{s})$ | Inverse sine $f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_s(\bar{s})\sin sx\,ds$ |

## UV PHYSICS ACADEMY
### MATHEMATICAL PHYSICS

MP(1)

ECTOR CALCULUS:

calar product : $a\cdot b = |a||b|\cos\theta$ , vector product $a\times b = |a||b|\sin\theta\hat{n}$

r coplanar vector $[a\ b\ c] = 0$

ector triple product : $a\times(b\times c) = (a\cdot c)b - (a\cdot b)c$

calar product of four vectors : $(a\times b)\cdot(c\times d) = (a\cdot c)(b\cdot d) - (a\cdot d)(b\cdot c)$

ector product of four vectors : $(a\times b)\times(c\times d) = [abd]c - [abc]d$

eciprocal system of vectors : $(a\cdot a') = (b\cdot b') = (c\cdot c') = 1$

$' = \frac{b\times c}{[abc]}$ $b' = \frac{c\times a}{[abc]}$ $c' = \frac{a\times b}{[abc]}$

$\nabla r = \frac{\bar{r}}{r}$

$\nabla\cdot\bar{r} = 3$

$\nabla r^2 = 2\bar{r}$

$\nabla\cdot(r\bar{r}) = 4r$

$\nabla(1/r^2) = \frac{-2\bar{r}}{r^4}$

$\nabla\cdot(\bar{r}/r^3) = 4\pi\delta(r)$

$\nabla(1/r) = \frac{-\bar{r}}{r^3}$

$\nabla\cdot(\bar{r}/r^2) = 1/r^2$

$\nabla^2 r^n = n(n+1)r^{n-2}$ , $\nabla r^n = nr^{n-2}\bar{r}$

$\nabla\cdot(r^n\bar{r}) = (3+n)r^n$

$(fg) = f\nabla g + g\nabla f$

$(fA) = f\nabla\cdot A + A\cdot\nabla f$ ; $\nabla\cdot(\nabla\times A) = 0$

$\times(fA) = f\nabla\times A + (\nabla f)\times A$ ; $\nabla\times(\nabla f) = 0$

$\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A$

$(A\times B) = B\cdot(\nabla\times A) - A\cdot(\nabla\times B)$

$\times(A\times B) = A(\nabla\cdot B) - B(\nabla\cdot A) + (B\cdot\nabla)A - (A\cdot\nabla)B$

$(A\cdot B) = A\times(\nabla\times B) + (A\cdot\nabla)B + B\times(\nabla\times A) + (B\cdot\nabla)A$

Gauss divergence theorem : $\iint_s A\cdot ds = \iiint_v \operatorname{div} A\, dv$

Stokes theorem : $\iint_s \operatorname{curl} A\cdot ds = \int_c A\cdot dr$

Green's Theorem $\iiint(\Psi_1\nabla^2\Psi_2 - \Psi_2\nabla^2\Psi_1)dx\,dy\,dz$

$\iint(\Psi_1 \operatorname{grad}\Psi_2 - \Psi_2 \operatorname{grad}\Psi_1)\cdot ds$

## UV PHYSICS ACADEMY

MMP(3)

**Standard Integrals**

$\int xe^{ax}dx = e^{ax}\left(\frac{x}{a} - \frac{1}{a^2}\right)$ $\int b^{ax}dx = \frac{b^{ax}}{a\ln b}$ $(b>0)$

$\int\frac{1}{a+bx}dx = \frac{1}{b}\ln(a+bx)$ $\int\frac{1}{x(a+bx)}dx = -\frac{1}{a}\ln\frac{a+bx}{x}$

$\int\frac{x}{(x^2\pm a^2)^n}dx = \frac{-1}{2(n-1)(x^2\pm a^2)^{n-1}}$ $\int\frac{x}{x^2\pm a^2}dx = \frac{1}{2}\ln|x^2\pm a^2|$

$\int\frac{1}{(x^2\pm a^2)^{1/2}}dx = \ln\left|x + (x^2\pm a^2)^{1/2}\right|$

$\int\frac{1}{x(x^n+a)}dx = \frac{1}{an}\ln\left|\frac{x^n}{x^n+a}\right|$ $\int\frac{x}{(x^2\pm a^2)^{1/2}}dx = (x^2\pm a^2)^{1/2}$

Error function erf(x): $\frac{2}{\pi^{1/2}}\int_0^x \exp(-t^2)dt$

Complementary error fucntion: $\frac{2}{\pi^{1/2}}\int_x^\infty \exp(-t^2)dt$

Gamma function $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ $(x>0)$

$\int_{-\infty}^{\infty} x^n e^{-ax^2}dx = 0$ If n is odd $\int_{-\infty}^{\infty} e^{-ax^2}dx = \left(\frac{\pi}{a}\right)^{1/2}$

$\int_{-\infty}^{\infty} x^{2n}e^{-ax^2}dx = \sqrt{\pi}\frac{1.3.5....(2n-1)}{2^n a^{n+1/2}}$

$\int_0^1 x^p(1-x)^q dx = \frac{p!q!}{(p+q+1)!}$ (p, q int egers > 0)

**Series Expansions**

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ......$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + .....$ $\ln\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + ...\right)$

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$ $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ....$ $\sec(x) = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + ....$

$\csc(x) = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + ...$ $\tan(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + ...$ $\cot(x) = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} + ...$

$\sum_{i=1}^n i = \frac{n}{2}(n+1)$ $\sum_{i=1}^n i^3 = \frac{n^2}{4}(n+1)^2$ $\sum_{i=1}^n i^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$ $\sum_{i=1}^\infty\frac{(-1)^{i+1}}{2i-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ..... = \frac{\pi}{4}$

$\sum_{i=1}^n i^2 = \frac{n}{6}(n+1)(2n+1)$ $\sum_{i=1}^\infty\frac{1}{i^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + ..... = \frac{\pi^2}{6}$ $\sum_{i=1}^\infty\frac{(-1)^{i+1}}{i} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ..... = \ln 2$

## UV PHYSICS ACADEMY
### MATHEMATICAL PHYSICS

P(5)

sidues :

simple pole : $\operatorname{Res}_{z\to z_0} f(z) = \operatorname{Lt}_{z\to z_0}(z-z_0)f(z)$ ; If $f(z) = \frac{p(z)}{q(z)}$ & $q(z_0) = 0$ then $\operatorname{Res}_{z\to z_0} f(z) = \frac{p(z_0)}{q'(z_0)}$

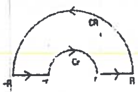
of order m>1: $\operatorname{Res}_{z\to z_0} f(z) = \frac{1}{(m-1)!}\operatorname{Lt}_{z\to z_0}\left[\frac{d^{m-1}}{dz^{m-1}}\{(z-z_0)^m f(z)\}\right]$ ; At infinity: $\operatorname{Res}_{z\to\infty} f(z) = \operatorname{Lt}_{z\to\infty}[-zf(z)]$

$\operatorname{Lt}_{z\to\infty}[-zf(z)]$ does not exist, Res f(z) = negative of coefficient of 1/z in expansion of f(z)

uchy Residue theorem : $\oint f(z)dz = 2\pi i\sum_{k=1}^n \operatorname{Res}_{z=z_k} f(z)$

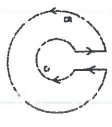
aluation of definite integrals a) Integral form $\int_0^{2\pi} f(\sin\theta\cos\theta)d\theta \to$ put $e^{i\theta} = z$

Integral of the form $\int_{-\infty}^{\infty} f(x)dx \to$ Can be solved as $\operatorname{Lt}_{R\to\infty}\int_{-R}^{R} f(x)dx$



Evaluation of infinite integral when integrand has pole on real axis :

re we delete the region indenting the contour.



Evaluation of integral involving many valued function :

Integral of the form $\int_0^\infty x^{\alpha-1}g(x)dx$ can be solved as $\int_0^\infty x^{\alpha-1}g(x)dx = \frac{2\pi i\sum R}{1 - e^{2\pi i\alpha}}$

## UV PHYSICS ACADEMY

MMP(7)

Equation of the form : $\frac{d^2y}{dx^2} + a_0\frac{dy}{dx} + a_1 y = F(x)$ , solution is $y = y_c + y_p$

$y_c = c_1 y_1(x) + c_2 y_2(x)$, $y_P = Ay_1(x) + By_2(x)$

$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$ ; $A = -\int\frac{F(x)y_2(x)}{W(y_1, y_2)}dx$ $B = \int\frac{F(x)y_1(x)}{W(y_1, y_2)}dx$

Legendre's linear equation: $(ax+b)^n\frac{d^n y}{dx^n} + P_0(ax+b)^{n-1}\frac{d^{n-1}(y)}{dx^{n-1}} + ... + P_n(y) = Q(x)$

put $(ax + b) = e^t$ we can reduce it into linear differential equation.

Linear differential equation with variable coefficients: $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$

Finding a part of complimentary function (C.F):

1) If $1 + P + Q = 0$ then $e^x$ is a part of C.F. ; 2) If $1 - P + Q = 0$ then $e^{-x}$ is a part of C.F

3) If $a^2 + Pa + Q = 0$ then $e^{ax}$ is a part of C.F ; 4) If $P + Qx = 0$ then x is a part of C.F

5) If $2 + 2Px + Qx^2 = 0$ then $x^2$ is part of C.F; 6) If $m(m-1) + Pmx + Qx^2 = 0$, $x^m$ is part of C.F.

**Laplace Transform :** i) $L\left[\int_0^t F(x)dx\right] = \frac{1}{s}L[F(t)]$ ii) $L[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}L[F(t)]$

iii) $L\left[\frac{F(t)}{t}\right] = \int_s^\infty F(x)dx$ iv) $L\left[\int_0^t\frac{\sin x}{x}dx\right] = \frac{1}{s}\tan^{-1}\frac{1}{s}$ iv) $L\left[\int_t^\infty\frac{\cos x}{x}dx\right] = \frac{1}{2s}\log(s^2+1)$, vi) $L[\delta(t-a)] = e^{-as}$

## UV PHYSICS ACADEMY
### Fourier Series:

MMP(9)

Def. : $f(x) = \frac{a_0}{2} + \sum_{n=1}^\infty a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^\infty b_n\sin\frac{n\pi x}{L}$ ; Complex form: $f(x) = c_0 + \sum_{n=1}^\infty c_n e^{in\pi x/L} + \sum_{n=1}^\infty c_{-n}e^{-in\pi x/L}$

$a_0 = \frac{1}{L}\int_c^{c+2L} f(x)dx$, $a_n = \frac{1}{L}\int_c^{c+2L} f(x)\cos\frac{n\pi x}{L}dx$, $b_n = \frac{1}{L}\int_c^{c+2L} f(x)\sin\frac{n\pi x}{L}dx$

$c = (0,\ 2L) \to a_0 = \frac{1}{L}\int_0^{2L} f(x)dx$, $a_n = \frac{1}{L}\int_0^{2L} f(x)\cos\frac{n\pi x}{L}dx$, $b_n = \frac{1}{L}\int_0^{2L} f(x)\sin\frac{n\pi x}{L}dx$

$c = (-L,\ L) \to a_0 = \frac{1}{L}\int_{-L}^{+L} f(x)dx$, $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx$, $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx$

| If $f(-x) = f(x)$ then, | if $f(-x) = -f(x)$ then, |
|---|---|
| $f(x) = \frac{a_0}{2} + \sum_{n=1}^\infty a_n\cos\frac{n\pi x}{L}$ <br> $a_0 = \frac{2}{L}\int_0^L f(x)dx$ , $a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$ <br> This is a cosine series | $f(x) = \sum_{n=1}^\infty b_n\sin\frac{n\pi x}{L}$ <br> $b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$ <br> This is a sine series |

**Dirichlets condition :**

Any function f(x) can be written as fourier series

## UV PHYSICS ACADEMY

MMP(11)

Linearity property : $F[af(x) + bg(x)] = a\,F(s) + bG(s)$ $F[f(ax)] = \frac{1}{a}F\left(\frac{s}{a}\right)$, $a\neq 0$

$F[f(x-a)] = e^{isa}F(s)$; $F\left[\frac{d^n f}{dx^n}\right] = (-is)^n F[f(x)]$; Convolution theorem: $F[f(x)\times g(x)] = F[f(x)]\,F[g(x)]$

Modulation theorem : $F[f(x)\cos ax] = \frac{1}{2}[F(s+a) + F(s-a)]$

Note : Fourier cosine transform : $F_c[f(x)\sin ax] = \frac{1}{2}[F_s(s+a) - F_s(s-a)]$

Fourier sine transform :

$F_s[f(x)\cos ax] = \frac{1}{2}[F_s(s+a) + F_s(s-a)]$

Fourier sine transform :

$F_s[f(x)\sin ax] = \frac{1}{2}[F_c(s-a) - F_c(s+a)]$

Relation between Fourier and Laplace transformations If $f(t) = \begin{cases} e^{-xt}\ g(t) & t>0 \\ 0 & t<0 \end{cases}$ then $F[f(t)] = L[g(t)]$

**Series:**

Infinite series : $\sum u_n = u_1 + u_2 + ...u_n + ...\infty$ $u_1, u_2$... be real number

Sum of first n terms : $s_n = u_1 + u_2 + ... + u_n = \sum u_n$ ;

As $n\to\infty$

- $s_n \to$ finite limit, $\sum u_n$ is convergent
- $s_n \to \pm\infty$, $\sum u_n$ is divergent
- $s_n$ does not tend to unique limit, $\sum u_n$ is oscillatory (or) non-convergent

Geometric series :

$1 + r + r^2 + ...\infty$

converges if $|r| < 1$

diverges if $r \geq 1$

oscillates if $r \leq -1$

## UV PHYSICS ACADEMY
### QUANTUM MECHANICS

**QM(1)**

**DeBroglie Wavelength :**

| Charged particles | |
|---|---|
| Non relativistic ($E_k < E_0$) | Relativistic case ($E_k \geq E_0$) |
| $\lambda_{NR} = \dfrac{h}{\sqrt{2mE_k}}\,\mathring{A}$ | $\lambda_R = \dfrac{hc}{\sqrt{E_k^2 + 2E_k E_0}}\,\mathring{A}$ |

$E_k$ = kinetic energy
$E_0$ = rest mass energy
T = absolute temperature
K = boltzmann constant
λ = compton wavelength
$\gamma = 1/\sqrt{1-(v^2/c^2)}$
k = magnitude of wave vector
w = angular frequency
$v_p$, $v_g$ = phase, group velocity
λr = Rel.debrogli wavelength

$\lambda_e = \dfrac{12.26}{\sqrt{V}}\mathring{A} = \sqrt{\dfrac{150}{V}}\mathring{A}$, $\lambda_{photon} = \dfrac{hc}{E(eV)}\mathring{A}$, $\lambda_p = \dfrac{0.286}{\sqrt{V}}\mathring{A}$

$\lambda_R = \dfrac{\lambda_c}{\sqrt{\gamma^2-1}}\mathring{A}$, $\lambda_\alpha = \dfrac{0.101}{\sqrt{V}}\mathring{A}$

Neutral particles in 3D $\lambda = \dfrac{h}{\sqrt{3mKT}}$, $E = \dfrac{3}{2}KT$ ; Neutral gas : $\lambda = \dfrac{h}{\sqrt{2mKT}}$, $E = KT$

**Group and phase velocity :** $v_p = \dfrac{\omega}{k} = \dfrac{E}{p}$, $v_g = \dfrac{d\omega}{dk} = \dfrac{dE}{dp}$ ; $v_g = v_p + k\dfrac{dv_p}{dk}$, $v_g = v_p - \lambda\dfrac{dv_p}{d\lambda}$

for relativistic particle $v_p v_g = c^2$ ; for free particle $v_g = 2v_p$

Medium

- Non dispersive ($dv_p/d\lambda = 0$): $v_g = v_p$
- Dispersive ($dv_p/d\lambda \neq 0$)
  - $v_g = v_p - \lambda\dfrac{dv_p}{d\lambda}$ (normal dispersion)
  - $v_g = v_p + \lambda\dfrac{dv_p}{d\lambda}$ (Anamolous dispersion)

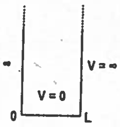
## UV PHYSICS ACADEMY

**QM(3)**

**A free particle in 1D box**

$V(x) = 0$ ; $0 \leq x \leq L$

$V(x) = \infty$ ; $x < 0$ & $x > L$



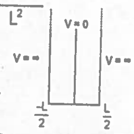

V(x) = potential energy
$\psi_n$ = wave function of $n^{th}$ state
$E_n$ = energy of $n^{th}$ state
L = box length

$\psi_n = \sqrt{\dfrac{2}{L}}\sin\dfrac{n\pi x}{L}$, $E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$, n = 1, 2, 3...

**Density of states :** $\dfrac{dn}{dE} = \dfrac{n}{2E}$

**Expectation values :** $\langle x\rangle = \dfrac{L}{2}$, $\langle x^2\rangle = \dfrac{L^2}{3} - \dfrac{L^2}{2n^2\pi^2}$, $\langle p_x\rangle = 0$, $\langle p_x^2\rangle = \dfrac{n^2\pi^2\hbar^2}{L^2}$

A free particle in 1D box (symmetric)




$\psi_n = \sqrt{\dfrac{2}{L}}\cos\dfrac{n\pi x}{L}$, n = 1, 3, 5..., $\psi_n = \sqrt{\dfrac{2}{L}}\sin\dfrac{n\pi x}{L}$, n = 2, 4, 6..., $E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$

**A free particle in 3D box:** $\psi_n = \sqrt{\dfrac{8}{abc}}\sin\left(\dfrac{n_x\pi x}{a}\right)\sin\left(\dfrac{n_y\pi y}{b}\right)\sin\left(\dfrac{n_z\pi z}{c}\right)$ ; $E = \dfrac{h^2}{8m}\left[\dfrac{n_x^2}{a^2} + \dfrac{n_y^2}{b^2} + \dfrac{n_z^2}{c^2}\right]$,

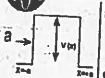
for cubic box a = b = c, $E = \dfrac{h^2}{8ma^2}\left[n_x^2 + n_y^2 + n_z^2\right]$

## UV PHYSICS ACADEMY

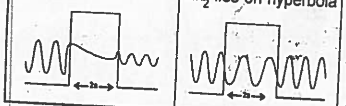
**QM(5)**

**Potential Barrier**

$V(x) = 0$, $x < -a$
$= V$, $-a < x < a$
$= 0$, $x > a$



| E < V | E > V |
|---|---|
| $T \approx \dfrac{16E(V_0 - E)}{V_0^2}e^{-2k_2 a}$ | $\dfrac{1}{T} = 1 + \dfrac{1}{4}\dfrac{V^2}{E(E-V)}\sin^2 2k_2 a$ |
| The energy spectrum is abounded continum with $ak_1$ & $ak_2$ lies on a circle | The energy spectrum is a un bounded continuum with $ak_1$ & $ak_2$ lies on hyperbola |

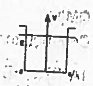


$k_1^2 = \dfrac{2mE}{\hbar^2}$ ; $k_2^2 = \dfrac{2m(E-V)}{\hbar^2}$, E > V

$k_2^2 = \dfrac{2m(V-E)}{\hbar^2}$, E < V ;

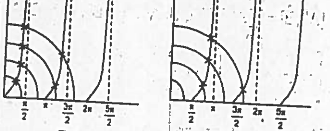
T = transmission coefficient

**Bound states**



Bound states

- Symmetric: $k_1 a = k_2 a \tan k_2 a$
- Antisymmetric: $-k_1 a = k_2 a \cot k_2 a$



$0 < r < \dfrac{\pi}{2}$ #SBS=1 $0 < r < \dfrac{\pi}{2}$ #ABS=0

$\dfrac{\pi}{2} < r < \dfrac{3\pi}{2}$ #SBS=3 $\dfrac{\pi}{2} < r < \dfrac{3\pi}{2}$ #ABS=2

$(k_1 a)^2 + (k_2 a)^2 = r^2 = 2mVa^2/\hbar^2$

$(n-1)^2\dfrac{\pi^2\hbar^2}{8ma^2} < V < \dfrac{n^2\pi^2\hbar^2}{8ma^2}$

Total number of bound state = n,
SBS ,ABS = Symmetric,antisymmetric bound state respectively.

## UV PHYSICS ACADEMY

**QM(7)**

**Linear Harmonic Oscillator (LHO)**

$V(x) = \dfrac{1}{2}kx^2 = \dfrac{1}{2}m\omega^2x^2$, $|x| > 0$ ; $\psi_n(x) = \sqrt{\dfrac{\alpha}{2^n n!\sqrt{\pi}}}e^{-\frac{1}{2}\alpha^2x^2}H_n(\alpha x)$ ; $E_n = \left(n + \dfrac{1}{2}\right)\hbar\omega$, n=0, 1, 2...

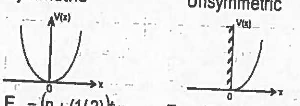
$H_n(\alpha x) = (-1)^n e^{\frac{1}{2}\alpha^2x^2}\dfrac{d^n}{dx^n}e^{-\frac{1}{2}\alpha^2x^2}$ ; **Ladderoperators:** $\hat{a}^+ = \dfrac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{x} - i\hat{p}_x)$, $\hat{a} = \dfrac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{x} + i\hat{p}_x)$

$\hat{a}^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$; $\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$ ; $N = \hat{a}^+\hat{a}$, $\hat{H} = \hbar\omega\left(\hat{a}^+\hat{a} + \dfrac{1}{2}\right) = \hbar\omega\left(\hat{a}\hat{a}^+ - \dfrac{1}{2}\right)$

$\langle(\hat{a}^+ + \hat{a})^n\rangle = 0$, if n = 1, 3, 5... ; $\langle(\hat{a}^+ + \hat{a})^2\rangle = 2n+1$ ; $\langle(\hat{a}^+ + \hat{a})^4\rangle = 6n^2 + 6n + 3$

$[\hat{N}, \hat{a}] = -\hat{a}$, $[\hat{N}, \hat{a}^+] = \hat{a}^+$ ; $[\hat{a}^+, \hat{H}] = -\hbar\omega\hat{a}^+$ ; $[\hat{a}, \hat{H}] = \hbar\omega\hat{a}$ ; $H_n(\alpha x)$ = Hermite polynomial $\alpha^2 = \dfrac{m\omega}{\hbar}$

LHO



| Symmetric | Unsymmetric |
|---|---|
| $E_n = (n + (1/2))\hbar\omega$ | $E_n = (2n + (3/2))\hbar\omega$ |
| n = 0,1,2... | n = 0,1,2... |
| $V(x) = (1/2)kx^2$, \|x\|>0 | $V(x) = \infty$, x ≤ 0 |
| | $V(x) = (1/2)kx^2$, x > 0 |

Isotropic harmonic oscillator

2 D ($\omega_x = \omega_y = \omega$): $H = \dfrac{p_x^2}{2m} + \dfrac{p_y^2}{2m} + \dfrac{1}{2}k(x^2 + y^2)$

$E = \hbar\omega(n_x + n_y + 1)$

3 D ($\omega_x = \omega_y = \omega_z = \omega$) : $E = \hbar\omega(n_x + n_y + n_z + 3/2)$

$H = \dfrac{p_x^2}{2m} + \dfrac{p_y^2}{2m} + \dfrac{p_z^2}{2m} + \dfrac{1}{2}k(x^2 + y^2 + z^2)$

Degeneracy of $n^{th}$ excited state = $(N+1)(N+2)/2$

$N = 2n + l$, n = 0, 1, 2...

## UV PHYSICS ACADEMY

**QM(9)**

**Hydrogen Atom**

| | |
|---|---|
| (1s) $\psi_{100} = \dfrac{1}{\sqrt{\pi}}\left(\dfrac{1}{a_0}\right)^{3/2}e^{-r/a_0}$ | (2s) $\psi_{200} = \dfrac{1}{\sqrt{\pi}}\left(\dfrac{1}{2a_0}\right)^{3/2}\left(1 - \dfrac{r}{2a_0}\right)e^{-r/2a_0}$ ; (2p) $\psi_{210} = \dfrac{1}{\sqrt{\pi}}\left(\dfrac{1}{2a_0}\right)^{5/2}re^{-r/2a_0}\cos\theta$ |
| (2p) $\psi_{21,\pm1} = \mp\dfrac{1}{8\sqrt{\pi}}\left(\dfrac{1}{a_0}\right)^{5/2}re^{-r/2a_0}\sin\theta e^{\pm i\varphi}$ | (3s) $\psi_{300} = \dfrac{1}{81\sqrt{3\pi}}\left(\dfrac{1}{a_0}\right)^{3/2}\left(27 - \dfrac{18r}{a_0} + \dfrac{2r^2}{a_0^2}\right)e^{-\frac{r}{3a_0}}$ |
| (3p) $\psi_{31\pm1} = \dfrac{1}{81\sqrt{3\pi}}\left(\dfrac{1}{a_0}\right)^{3/2}\left(6 - \dfrac{r}{a_0}\right)\dfrac{r}{a_0}e^{-\frac{r}{3a_0}}\sin\theta e^{\pm i\phi}$ | $a_0$ (0.53 Å) = Bohr radius, r = radius of hydrogen atom |
| $\langle r^2\rangle = \dfrac{n^2}{2}\left[5n^2 + 1 - 3\ell(\ell+1)\right]a_0^2$ | $\langle r\rangle = \dfrac{1}{2}\left[3n^2 - \ell(\ell+1)\right]a_0$ ; $\left\langle\dfrac{1}{r^3}\right\rangle = \dfrac{2}{n^3\ell(\ell+1)(2\ell+1)a_0^3}$ |
| $\left\langle\dfrac{1}{r^4}\right\rangle = \dfrac{4\left[3n^2 - \ell(\ell+1)\right]}{n^5\ell(\ell+1)(2\ell+1)\left[(2\ell+1)^2 - 4\right]a_0^4}$ | $\left\langle\dfrac{1}{r}\right\rangle = \dfrac{1}{n^2a_0}$ ; $\left\langle\dfrac{1}{r^2}\right\rangle = \dfrac{2}{n^3(2\ell+1)a_0^2}$ |
| $Y_{0,0} = \dfrac{1}{\sqrt{4\pi}}$ ; $Y_{1,0} = \sqrt{\dfrac{3}{4\pi}}\cos\theta$ | $Y_{2,0} = \sqrt{\dfrac{5}{16\pi}}(3\cos^2\theta - 1)$ ; $Y_{1,\pm1} = \mp\sqrt{\dfrac{3}{8\pi}}(\sin\theta)e^{\pm i\phi}$ |
| $Y_{2\pm1} = \mp\sqrt{\dfrac{15}{32\pi}}(\sin 2\theta)e^{\pm i\phi}$ ; $Y_{2,\pm2} = \sqrt{\dfrac{15}{32\pi}}(\sin^2\theta)e^{\pm 2i\phi}$ | Coulomb degeneracy = $\sum_{\ell=0}^{n-1}(2\ell+1) = n^2$. Including spin degeneracy = $2n^2$. |

## UV PHYSICS ACADEMY

**QM(11)**

Scattering :Assymptotic form : $\psi = A\left[e^{ikz} + f(\theta)\dfrac{e^{ikr}}{r}\right]$ ; $\dfrac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2$ ; $\sigma = |f(0)|^2$

Born approximation: $f(\theta) = \dfrac{-2\mu}{\hbar^2}\int_0^\infty r'V(r')\sin(qr')\,dr'$ ;

$\sigma = \dfrac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell$,

Partial wave analysis : $f(\theta) = \sum_{\ell=0}^{\infty}\dfrac{(2\ell+1)}{k}\sin\delta_\ell e^{i\delta_\ell}P_\ell(\cos\theta)$

Optical theorem $\sigma = \dfrac{4\pi}{k}\mathrm{Im}f(0)$

Breit Wigner formula : $\sigma = \dfrac{4\pi}{k^2}\dfrac{\Gamma^2/4}{(E - E_r)^2 + \Gamma^2/4}$

Properties Pauli spin matrices

$T_r(\sigma_i) = 0$ $[\sigma_i, \sigma_j] = 2i\in_{ijk}\sigma_k$ $\in_{ijk} = \begin{cases}+1 \text{ for cyclic}\\ -1 \text{ for anti cyclic}\\ 0 \text{ otherwise}\end{cases}$

$T_r(\sigma_i, \sigma_j) = 2\delta_{ij}$ $\sigma_+\sigma_- = 2(1 + \sigma_z)$

$e^{i\theta\sigma} = I\cos\theta + i\sigma\sin\theta$ $[\hat{\sigma}, \vec{A}.\hat{\sigma}] = 2i\vec{A}\times\hat{\sigma}$

$\sigma_+ = \sigma_x + i\sigma_y$; $\sigma_- = \sigma_x - i\sigma_y$ $\sigma_\pm = \sigma_x \pm i\sigma_y$ $\sigma_+^2 = 0 = \sigma_-^2$

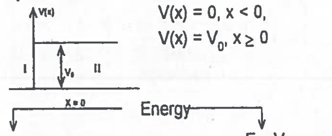
$(\hat{\sigma}.\vec{A})(\hat{\sigma}.\vec{B}) = \vec{A}.\vec{B} + i\hat{\sigma}.(\vec{A}\times\vec{B})$ $\sigma_+\sigma_- = 2(1 + \sigma_z)$

$f(\theta,\phi)$ = scattering amplitude
$Af(\theta,\phi)\dfrac{e^{ikr}}{r}$ = out going wave
$Ae^{ikz}$ = incident wave
$d\sigma/d\Omega$ =differential scattering cross section
σ = total cross section
k = wave vector
Imf(0) = imaginary part of scattering amplitude
Γ = width of resonant curve
$E_r$=resonant energy

## UV PHYSICS ACADEMY

QM(4)

**Step Potential**



$V(x) = 0,\ x < 0,$
$V(x) = V_0,\ x \geq 0$

R = reflection coefficient
T = transmission coefficient

$k_1^2 = \frac{2mE}{\hbar^2} (E \leq V_0, E > V_0)$

$k_2^2 = \frac{2m(V_0 - E)}{\hbar^2} (E < V_0)$

$k_2^2 = \frac{2m(E - V_0)}{\hbar^2} (E > V_0)$

$k_2^2 = \frac{2m(E + V)}{\hbar^2}$

Energy

$E \leq V_0$

$$\frac{\partial^2\psi_1}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1 = 0$$

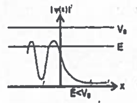
$$\frac{\partial^2\psi_2}{\partial x^2} - \frac{2m(V_0 - E)}{\hbar^2}\psi_2 = 0$$

$E > V_0$

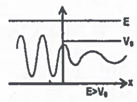
$$\text{I}: \frac{\partial^2\psi_1}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1 = 0$$

$$\text{II}: \frac{\partial^2\psi_2}{\partial x^2} + \frac{2m(E - V_0)}{\hbar^2}\psi_2 = 0$$

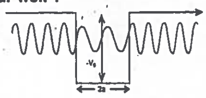
$R = 1,\ T = 0$



$$R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2;\ T = \frac{4k_1k_2}{(k_1 + k_2)^2}$$



**Potential well :**



$V(x) = -V_0,\ |x| < a, \quad = 0,\ |x| > a$

<u>E > V</u> $\quad \frac{1}{T} = 1 + \frac{1}{4}\left[\frac{V^2}{E(E+V)}\right]\sin^2 2k_2a$

## UV PHYSICS ACADEMY

QM(2)

Wavefunction and Schrodinger's equation

$\psi(x)$ and $\frac{d\psi(x)}{dx}$ must be: finite, single valued, continuous

$\frac{1}{\psi}\frac{\partial\psi}{\partial x}$ is continuous

$\hat{H}$ = hamiltonian of the system
E = energy eigen value
$\psi$ = wave function of the system
J = current density
$\psi^*$ = complex conjugate of $\psi$
I.P = imaginary part
Re = real part
$\rho$ = charge density = $\psi^*\psi$

Schrodinger equation

Time independent: $\hat{H}\psi = E\psi \left(\because \hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + V\right)$

Time dependent: $i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$

Current density : $J = \frac{\hbar}{2im}\left[\psi^*\nabla\psi - \psi\nabla\psi^*\right]$, $J = \frac{\hbar}{m}\left[\text{I.P of } \psi^*\nabla\psi\right]$, $J = \frac{1}{m}\left[\psi^*\hat{P}\psi + (\psi^*\hat{P}\psi)^*\right]$

Equation of continuity : $\nabla\cdot J + \frac{\partial\rho}{\partial t} = 0$

**Operators :** Linear : $\hat{O}(a\psi + b\phi) = a\hat{O}\psi + b\hat{O}\phi$ ; Hermitian : $\hat{A}^+ = A$, $\langle\psi|\hat{A}\psi\rangle = \langle\hat{A}\psi|\psi\rangle$

Projection : $P_\psi = |\psi\rangle\langle\psi|$ ; Completeness rule : $\sum_\psi |\psi\rangle\langle\psi| = I$ ; momentum : $-i\hbar\nabla$

Kinetic energy : $\frac{-\hbar^2}{2m}\nabla^2 + V$ ; Triangle inequality: $\sqrt{\langle\psi+\varphi|\psi+\varphi\rangle} \leq \sqrt{\langle\psi|\psi\rangle} + \sqrt{\langle\varphi|\varphi\rangle}$

Parity : $\hat{P}\psi(x) = \pm\psi(-x)$ ; $\hat{x} = i\hbar\frac{\partial}{\partial p_x}$ Cauchy-Schwartz inequality: $|\langle\varphi|\psi\rangle|^2 \leq \langle\psi|\psi\rangle\langle\varphi|\varphi\rangle$

## UV PHYSICS ACADEMY

QM(8)

Angular momentum :

| | | | | |
|---|---|---|---|---|
| $\bar{L} = \bar{r} \times \bar{p}$ | $\hat{L}_+\lvert\ell,m\rangle = \hbar\sqrt{(\ell-m)(\ell+m+1)}\,\lvert\ell,m+1\rangle$ | | $\hat{L}^2\lvert\ell,m\rangle = \hbar^2\,\ell(\ell+1)\lvert\ell,m\rangle$ | |
| $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$ | $\hat{L}_-\lvert\ell,m\rangle = \hbar\sqrt{(\ell+m)(\ell-m+1)}\,\lvert\ell,m-1\rangle$ | | $\hat{L}_z\lvert\ell,m\rangle = m\hbar\,\lvert\ell,m\rangle$ | |
| $\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y$ | $\langle\hat{L}_x^2\rangle = \langle\hat{L}_y^2\rangle = \langle\hat{L}^2 - \hat{L}_z^2\rangle = \frac{\hbar^2}{2}\left[\ell(\ell+1) - m^2\right]$ | | $[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z$ | $[\hat{L}_x, \hat{L}_+] = -\hbar\hat{L}_z$ |
| $\hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z$ | $[\hat{p}_x, \hat{L}^2] = i\hbar[\hat{L}_y\hat{p}_z + \hat{p}_z\hat{L}_y - \hat{L}_z\hat{p}_y - \hat{p}_y\hat{L}_z]$ | | $[\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x$ | $[\hat{L}_y, \hat{L}_+] = -i\hbar\hat{L}_z$ |
| $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$ | $[\hat{x}, \hat{L}^2] = i\hbar[\hat{z}\hat{L}_y + \hat{L}_y\hat{z} - \hat{L}_z\hat{y} - \hat{y}\hat{L}_z]$ | | $[\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y$ | $[\hat{L}_z, \hat{L}_-] = -\hbar\hat{L}_-$ |
| $\langle\hat{L}_x\rangle = \langle\hat{L}_y\rangle = 0$ | $\hat{L}_+ = \hat{L}_x + i\hat{L}_y$ ; $\hat{L}_- = \hat{L}_x - i\hat{L}_y$ | $\hat{L}_x = \frac{1}{2}(\hat{L}_+ + \hat{L}_-)$ ; $\hat{L}_y = \frac{1}{2i}(\hat{L}_+ - \hat{L}_-)$ | $[\hat{L}_i, \hat{L}_j] = i\hbar\epsilon_{ijk}\hat{L}_k$ | $[\hat{L}_z, \hat{L}_+] = \hbar\hat{L}_+$ |
| Spherical Harmonics | | | | |
| $\hat{L}_x = i\hbar\left[\sin\phi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right]$ | $\hat{L}_y = i\hbar\left[-\cos\phi\frac{\partial}{\partial\theta} + \cot\theta\sin\phi\frac{\partial}{\partial\phi}\right]$ | | $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$ | |
| $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$ | | | $\hat{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$ | |
| $\hat{L}_+ = \hbar e^{i\phi}\left[\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right]$ | $\hat{L}_- = -\hbar e^{-i\phi}\left[\frac{\partial}{\partial\theta} - i\cot\theta\frac{\partial}{\partial\phi}\right]$ | | $\varepsilon_{ijk}$ = 1 for even permutation; = −1 for odd permutation; = 0 if any two are equal | |

## UV PHYSICS ACADEMY

QM(6)

<K> = Avg. value of kinetic energy; <V> = Avg. value of potential energy; n = integer

Commutators :

$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ ; $[\hat{A}, \hat{B}] = -[\hat{B}, \hat{A}]$ ; $[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$ ; $[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$

$[\hat{A}, [\hat{B}, \hat{C}]] + [\hat{B}, [\hat{C}, \hat{A}]] + [\hat{C}, [\hat{A}, \hat{B}]] = 0$ ; $e^{\hat{A}}e^{\hat{B}} = e^{\hat{A} + \hat{B} + \frac{[\hat{A}, \hat{B}]}{2}}$ ; $[f(\hat{A}), \hat{B}] = [\hat{A}, \hat{B}]\, f(\hat{A})$

$[\hat{A}^n, \hat{B}] = \sum_{j=0}^{n-1} A^{n-j-1}[A, B]A^j$ ; $[\hat{A}, \hat{B}^n] = \sum_{j=0}^{n-1}\hat{B}^j[\hat{A}, \hat{B}]\hat{B}^{n-j-1}$ ; $[\hat{x}, \hat{p}_x] = [\hat{y}, \hat{p}_y] = [\hat{z}, \hat{p}_z] = i\hbar$

$\left[x, \frac{\partial}{\partial x}\right] = -1$ ; $\left[f(x), \frac{\partial}{\partial x}\right] = -\frac{\partial}{\partial x}f(x)$ ; $e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A}, \hat{B}] + \frac{1}{2!}[\hat{A}, [\hat{A}, \hat{B}]] + \ldots$

Uncertanity Relation: $\Delta\hat{A} = \sqrt{\langle A^2\rangle - \langle\hat{A}\rangle^2}$ ; $\Delta\hat{A}\,\Delta\hat{B} = \frac{1}{2i}\langle[\hat{A}, \hat{B}]\rangle$ ;

A particle in 1 D box: $\Delta\hat{x} = L\sqrt{\frac{\pi^2 - 6}{12\pi}}$ ; $\Delta\hat{p}_x = \frac{n\pi\hbar}{L}$ ; LHO: $\Delta\hat{x}\ \Delta\hat{p}_x \geq (2n+1)\frac{\hbar}{2}$

For Gaussian wave packet $\Delta\hat{x}\ \Delta\hat{p}_x = \frac{\hbar}{2}$ Expectation value : $\langle\hat{o}\rangle = \frac{\langle\psi|\hat{O}|\psi\rangle}{\langle\psi|\psi\rangle}$ ;

Equn of motion: $\frac{d}{dt}\langle\hat{O}\rangle = \frac{1}{i\hbar}\langle[\hat{O}, \hat{H}]\rangle + \frac{\partial}{\partial t}\langle\hat{O}\rangle$ ; Eherenfest theorem: $\frac{d}{dt}\langle\hat{r}\rangle = \frac{\langle\hat{p}\rangle}{m}$, $\frac{d}{dt}\langle\hat{p}\rangle = -\nabla V$

Virial theorem : $F \propto r^n$ (or) $V \propto r^{n+1}$ ; $\langle K\rangle = \frac{n+1}{2}\langle V\rangle$ ; $\langle K\rangle = \frac{1}{2}\left\langle r\frac{\partial V}{\partial r}\right\rangle$

If $[\hat{A}, \hat{B}] = 0$ then they commute each other & $\hat{A}$ , $\hat{B}$ can be measured accurately, have same eigen functions

## UV PHYSICS ACADEMY

QM(12)

**Perturbation Theory**

Time independent perturbation theory : $\hat{H} = \hat{H}_0 + \hat{H}_p$

non degenerate perturbation : $\hat{H}_0|\phi_n\rangle = E_n^{(0)}|\phi_n\rangle$

| Parameter | Non degenerate perturbation | Degenerate perturbation $\lvert H - EI\rvert = 0$ |
|---|---|---|
| $E_n$ | $E_n^{(1)} = \langle\psi^{(0)}\lvert\hat{H}_p\rvert\psi^{(0)}\rangle$ | $E_\pm = \left(\frac{H_{11} + H_{22}}{2}\right) \pm \left[\left(\frac{H_{11} - H_{22}}{2}\right)^2 + \lvert H_{12}\rvert^2\right]^{1/2}$ |
| $\psi_n$ | $\lvert\bar{\psi}_n^{(1)}\rangle = \sum_{k\neq i}\frac{\lvert\langle k\lvert H\rvert i\rangle\rvert}{E_i^{(0)} - E_k^{(0)}}\lvert k\rangle$ | $\psi = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ $a_1 = \frac{H_{12}}{\sqrt{D}}$ & $a_2 = \frac{E - H_{11}}{\sqrt{D}}$ $D = (H_{11} - E)^2 + \lvert H_{12}\rvert^2$ |

Second order correction to energy : $E_n^{(2)} = \sum_{m\neq n}\frac{|\langle\phi_m|\hat{W}|\phi_n\rangle|^2}{E_n^{(0)} - E_m^{(0)}}$

WKB method : It is useful for a system with slowly varying potential.
It is semiclassical approximation .

Zeeman Effect : Perturbation $H^I = \frac{e}{2\mu}(\bar{B}\cdot\bar{L})$ represents the energy $(-\bar{M}\cdot\bar{B})$ of magnetic dipole with $\bar{M} = -\frac{e}{2\mu}\bar{L}$

First order energy correction term : $E_1 = \langle m|\frac{e}{2\mu}BL_z|m\rangle = \frac{eB}{2\mu}m\hbar$

$E_n$ = energy of $n^{th}$ state; $E_n^I$ = first order energy correction; $\psi_n$ = wave function of $n^{th}$ state

$\psi_n^I, \psi_\ell$ = first order correction to wave function; $H_0$ = non perturbed term;

$H_p$ = small perturbation; $\psi_n = \psi_n^0 + \psi_n^I$ ; $E_n = E_n^0 + E_n^I$ ;

## UV PHYSICS ACADEMY

QM(10)

Spin angular momentum :

For spin 1/2 particle:

$S_x = \frac{\hbar\sigma_x}{2} = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ ; $S_y = \frac{\hbar\sigma_y}{2} = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ ; $S_z = \frac{\hbar\sigma_z}{2} = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

Pauli spin matrices: $(\sigma_x, \sigma_y, \sigma_z)$
Eigen values = ±1

Eigen vector

<u>spin 1/2</u>

$\sigma_x$: $|x\uparrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ ; $|x\downarrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

$\sigma_y$: $|y\uparrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$ ; $|y\downarrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$

$\sigma_z$: $|z\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ ; $|z\downarrow\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

<u>spin 1</u>

$\sigma_x$: $|-1\rangle = \frac{1}{2}\begin{pmatrix}-1\\ \sqrt{2}\\ -1\end{pmatrix}$ ; $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\0\\1\end{pmatrix}$ ; $|1\rangle = \frac{1}{2}\begin{pmatrix}1\\ \sqrt{2}\\ 1\end{pmatrix}$

$\sigma_y$: $|-1\rangle = \frac{1}{2}\begin{pmatrix}1\\ -i\sqrt{2}\\ -1\end{pmatrix}$ ; $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix}$ ; $|1\rangle = \frac{1}{2}\begin{pmatrix}1\\ i\sqrt{2}\\ -1\end{pmatrix}$

$\sigma_z$: $|-1\rangle = \begin{pmatrix}-1\\0\\0\end{pmatrix}$ ; $|0\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}$ ; $|1\rangle = \begin{pmatrix}0\\0\\-1\end{pmatrix}$

<u>**For spin 1 particle :**</u>

$S_x = \frac{\hbar\sigma_x}{\sqrt{2}} = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$

$S_y = \frac{\hbar\sigma_y}{\sqrt{2}} = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}$

$S_z = \hbar\sigma_z = \hbar\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$

Eigen values of :

$\sigma_y = -\sqrt{2}(|-1\rangle), 0(|0\rangle), \sqrt{2}(|1\rangle)$

$\sigma_x, \sigma_z = -1(|-1\rangle), 0(|0\rangle), 1(|1\rangle)$,

# UV PHYSICS ACADEMY
## ATOMIC AND MOLECULAR PHYSICS

AMP(1)

**BOHR MODEL:**

| |
|---|
| m = mass of the electron |
| Z = atomic number |
| n = principle quantum number |
| R = Rydberg constant |
| e = electronic charge |
| μ = reduced mass |
| $E_{nH}$ = energy of hydrogen atom |
| $r_H$ = radius of hydrogen atom |

Quantisation condition : $L = mvr = n\hbar$

deBroglie hypothesis : $2\pi r_n = n\lambda$

Radius : $r_n = \frac{4\pi\epsilon_0 \hbar^2}{me^2}\frac{n^2}{Z} = 0.529\left(\frac{n^2}{Z}\right)\mathring{A}$

Velocity : $v_n = \frac{e^2}{4\pi\epsilon_0 \hbar}\frac{Z}{n} = 2.2\times10^6\left(\frac{Z}{n}\right) m/s$

Frequency : $f = \frac{me^4}{32\pi^3\epsilon_0^2\hbar^3}\frac{Z^2}{n^3}$ ; Energy : $E = \frac{-me^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{Z^2}{n^2} = -\frac{13.6Z^2}{n^2}eV$; $R = -E_l/ch$

Time period : $T_n = 1.51\times10^{-16}\left(\frac{n^3}{Z^2}\right)$ ; Wave number : $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$; $R = \frac{mZ^2e^4}{8\epsilon_0^2 h^3 c} = 1.097\times10^7 m^{-1}$

**Series :**

Lyman : $n_2 = 1, n_1 = 2, 3,..$ Balmer : $n_2 = 2, n_1 = 3, 4,..$ Paschen : $n_2 = 3, n_1 = 4, 5,..$

Bracket : $n_2 = 4, n_1 = 5, 6,..$ Pfund : $n_2 = 5, n_1 = 6, 7,..$ Humphry : $n_2 = 6, n_1 = 7, 8,..$

**Finite nuclear mass approximation :**

Radius : $r_n^I = mr_n/\mu$ ; Energy : $E_n^I = E_n\mu/m$ Rydberg constant : $R_z = R_d[1/(1+(m/M_z))]$

Positronium atom (Positron + electron): $E_{ep} = E_{nH}/2n^2 = -6.8/n^2 eV$ , $r_{ep} = 2r_H = 1.058n^2 A^0$

Muonic atom (p + μ⁻): $E_{\mu p} = (-186E_1/n^2)$; $E_1 = 13.6eV$ , $r_{\mu p} = (0.529n^2/186)\mathring{A}$

# UV PHYSICS ACADEMY

AMP(3)

Dipole transition

LS coupling

single electron transition: $\Delta\ell = \pm1, \Delta J = 0, \pm1$

$(0 \nrightarrow 0)$

multi electron atoms: $\Delta L = 0, \pm1, \Delta S = 0,$ $\Delta J = 0, \pm1$, $(0 \nrightarrow 0)$ $\Delta n =$ arbitrary(in one electron atom $\Delta L = 0$ not allowed)

For LS coupling : $J = |L - S|, |L - S|+1,....,|L+S|$

For an atom as a whole $\Delta J = 0, \pm1$ , $(0 \nrightarrow 0)$

j - j coupling: $\Delta j = 0, \pm1$ for jumping electron; $\Delta j = 0$ for all other electrons

Interaction

Electron - magnetic dipole

$\Delta E_{FS} = \frac{A}{2}[J(J+1) - L(L+1) - S(S+1)]$

F = J + I , A = fine structure coupling constant

Hyperfine

$\Delta E_{HFS} = \frac{A^I}{2}[F(F+1) - I(I+1) - J(J+1)]$

Lande's interval rule: $\Delta E_{FS} = A(J+1)$; $\Delta E_{HFS} = A^I(F+1)$

**Spectral terms**

**Equivalent electrons :**

1. Largest multiplicity lies lowest
2. For a given multiplicity, largest 'L' value lies lowest.
3. Less than half filled subshell : The level with lowest 'J' values lies lowest.
4. More than half filled subshel : The level with highest 'J' value lies lowest.

(Ground State spectralterms(G.S))

$^2He, {}^4Be, {}^{12}Mg, {}^{20}Ca, {}^{30}Zn, {}^{80}Hg \rightarrow {}^1S_0$

${}^6C \rightarrow {}^3P_0$ ${}^7N \rightarrow {}^4S_{3/2}$ ${}^8O \rightarrow {}^3P_2$

${}^{17}Cl \rightarrow {}^2P_{3/2}$ ${}^{24}Cr \rightarrow {}^7S_3$ ${}^{29}Cu \rightarrow {}^2S_{1/2}$

# UV PHYSICS ACADEMY

AMP(6)

| |
|---|
| v = vibrational quantum number |
| k = force constant |
| ε(0) = zero point energy |
| $x_e$ = anharmonic constant |
| $D_e$ = dissociation energy |

Vibrational spectra :

$E_\upsilon = (v + (1/2))\hbar\omega_{osc}$ Joules, $\omega_{osc} = \sqrt{k/\mu}$; $v = 0,1,2..$

$\epsilon_\upsilon = \left(v + \frac{1}{2}\right)\bar{\omega}_{osc} cm^{-1}$, $\bar{\omega}_{osc} = \frac{\omega_{osc}}{2\pi C} = \frac{1}{2\pi C}\sqrt{k/\mu}$; $\epsilon(0) = \frac{\omega_e}{2} - \frac{\omega_e x_e}{4}$

Maximum number of vibrational level below the dissociation limit

$v_{max} = (1/2x_e) - 1$; $D_e = \omega_e/4x_e - \omega_e x_e/4$

Vibrational - Rotational Spectra :

$E_{tot}(J, v) = E(J) + E(v) = \frac{J(J+1)\hbar^2}{2I} + \left(v + \frac{1}{2}\right)\hbar\omega$ ;

| Δv = ±1, ΔJ = | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| Branch = | O | P | Q | R | S |

Selection rules : (SHO): $\Delta J = \pm1$, $\Delta v = \pm1$ ; (Anharmonicoscillator): $\Delta J = \pm1, \pm2, \pm3....$, $\Delta v = \pm1$

$\Delta v = +1$ (absorption spectra), $\Delta v = -1$ (emission spectra)

ΔJ =
- −2 (O- branch), $\Delta\varepsilon_O(J, v) = \omega_e - 2\omega_e x_e - 2B(2J+3)$, $J = 0, 1, 2,...$
- − 1(P- branch), $\Delta\varepsilon_P(J, v) = \omega_e - 2\omega_e x_e - 2B(J+1)$, $J = 0, 1, 2,...$
- 0(Q-branch), $\Delta\varepsilon_Q = \omega_e - 2\omega_e x_e$
- 1(R-branch), $\Delta\varepsilon_R(J, v) = \omega_e - 2\omega_e x_e + 2B(J+1)$, $J = 0, 1, 3,...$
- 2(S- branch), $\Delta\varepsilon_S(J, v) = \omega_e - 2\omega_e x_e + 2B(2J+3)$, $J = 0, 1, 2,...$

$\bar{\omega}_x = \bar{\omega}_0 + 2Bm$, $m = 0, \pm1,...$, $m = 0$ (Q-branch) , m=1, 2,... (R-branch) , m = −1,−2,...(P-branch)

$\bar{\omega}_R = \omega_0 + 2B(J+1), J = 0, 1, ...$ ; $\bar{\omega}_P = \omega_0 - 2B(J+1), J = 0, 1, ...$ $\bar{\omega}_{PR}$(max Intense) $= 4B(J_{max}+1)$

$\bar{\omega}_R$(max Intense) $= \bar{\omega}_0 + 2B(J_{max}+1)$ ; $\bar{\omega}_P$(max Intense) $= \bar{\omega}_0 - 2B(J_{max}+1)$

# UV PHYSICS ACADEMY

AMP(7)

## Raman Effect

Raman scattering

Rayleigh line (coccur due to elastic scattering b/w photon and molecule): $\upsilon_{sc} = \upsilon_0$

Raman lines (occur due to inelastic scattering b/w photon and molecule)
- Anti stokes line: $\upsilon_{sc} > \upsilon_0$
- Stokes line: $\upsilon_{sc} < \upsilon_0$

Selection rule : $\Delta J = 2$ (stokes line)

$\Delta J = -2$ (anti-stokes line)

Pure vibrational Raman spectrum :

$\Delta\varepsilon(v) = \left(v + \frac{1}{2}\right)\omega_e - \left(v + \frac{1}{2}\right)^2\omega_e x_e cm^{-1}$,

$\Delta v = \pm1, \pm2..$

Intensity ratio : ($h\upsilon_m$ = molecular energy separation)

$\frac{I_S}{I_{AS}} = \left(\frac{\upsilon_0 - \upsilon_m}{\upsilon_0 + \upsilon_m}\right)^4 \exp\left(\frac{h\upsilon_m}{KT}\right)$

Stoke line has more intensity than anti-stokes line

Raman spectrum lies in visible region

Selection rule : $\Delta J = 0, \pm2$

**Pure rotational Raman Spectra :**

Change in rotational energy for Rayleigh line : $\Delta\varepsilon(J) = 0$, $\omega_{sc} = \omega_0$

For stokes line $\omega_s = \omega_0 - 2B(2J+3)cm^{-1}$, $J = 0, 1, 2...$

For anti-stokes line $\omega_{AS} = \omega_0 + 2B(2J-1)cm^{-1}$, $J = 2, 3, 4...$

Seperation between two consecutive rotational Raman line = 4B

Seperation between first stoke & first anti-stoke line = 12B

Speration between Rayleigh line & first Raman line = 6B

Total no. of symmetric spin state = (2I + 1) (I + 1)

Total no. of anti-symmetric spin state = (2I + 1) I.

| |
|---|
| v = vibrational quantum number |
| k = force constant |
| ε(0) = zero point energy |
| $x_e$ = anharmonic constai |
| $D_e$ = dissociation energ |

# UV PHYSICS ACADEMY
## PARTICLE PHYSICS

NP(13)

Spin 1/2 Baryon :

| Particle | Mass (MeV/c²) | Strangeness | Mean Life time (sec) | Typical decay | Quark content |
|---|---|---|---|---|---|
| p | 938.3 | 0 | Stable | - | uud |
| n | 939.6 | 0 | 886 | $p + e^- + \bar{\upsilon}_\mu$ | udd |
| $\Lambda^0$ | 1116 | −1 | $2.63\times10^{-10}$ | $p + \pi^-$ (or) $n + \pi^0$ | uds |
| $\Sigma^+$ | 1189 | −1 | $8.02\times10^{-11}$ | $p + \pi$(or) $n + \pi^+$ | uus |
| $\Sigma^0$ | 1193 | −1 | $7.4\times10^{-20}$ | $\Lambda^0 + \gamma$ | uds |
| $\Sigma^-$ | 1197 | −1 | $1.48\times10^{-10}$ | $n + \pi^-$ | dds |
| $\Xi^0$ | 1315 | −2 | $2.9\times10^{-10}$ | $\Lambda^0 + \pi^0$ | uss |
| $\Xi^-$ | 1321 | −2 | $1.64\times10^{-10}$ | $\Lambda^0 + \pi^-$ | dss |

Interactions :

| | C | P | T | CP | CPT |
|---|---|---|---|---|---|
| Strong | Y | Y | Y | Y | Y |
| E.M. | Y | Y | Y | Y | Y |
| Weak | N | N | N | N | Y |

## UV PHYSICS ACADEMY

AMP(4)

### Transition rules

$\Delta\ell = \pm1, \Delta J = 0, \pm1, \Delta S = 0, \Delta n =$ arbitrary

**Transition Rule**

- Normal Zeeman effect (Strong B): $\Delta m_\ell = 0, \pm1$ $(m_\ell = 0 \not\to m_\ell = 0$ If $\Delta\ell = 0)$
- Paschen back effect (Strong B, I = 0): $\Delta m_\ell = 0, \pm1, \Delta m_s = 0$
- Anomalous Zeeman effect: $\Delta m_j = 0, \pm1$ $(m_j = 0 \not\to m_j = 0$ If $\Delta J = 0)$
- Hyperfine: $\Delta F = 0, \pm1$ $(F=0 \not\to F=0)$

Magnetic field (B) =
- Strong and $S = \phi$ → Normal Zeeman effect
- Strong and $S \neq 0$ → Paschen Back effect
- Weak and $S \neq 0$ → Anamalous Zeeman effect

$\Delta E_{NZ} = \mu_B B_0 m_\ell$, $L = \frac{eB_0}{4\pi mc^2}$; No. of Paschen back level = (2L + 1) (2S + 1)

**Lambshift :** Due to the interaction between the electron and electromagnetic radiation field the energy state $2S_{1/2}$ is 0.035 $cm^{-1}$ higher than that of 2 $P_{1/2}$

**X-rays :** $h\upsilon_{max} = eV$, $\lambda_{min} = \frac{hc}{eV} = \frac{1234.5}{V}$ nm ; Intensity distribution : $I(\lambda) = K\left[\frac{C}{\lambda_{min}} - \frac{C}{\lambda}\right]\frac{1}{\lambda^2}$

Mosley's law : $\sqrt{\upsilon} = a(z-b)$, b =1 for K- shell, b = 7.4 for L - shell

Frequency : $\upsilon = \frac{C}{\lambda} = C\bar{\upsilon} = RC(z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$; Energy : $E = h\upsilon = RCh(z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\bar{\upsilon}$ = wave no.; V = potential; $\upsilon_{max}$ = maximum frequency; z = atomic no.; C = velocity of light

## UV PHYSICS ACADEMY

AMP(2)

Selection rules :

| | Electric dipole | Magnetic dipole | Electric quadrapole |
|---|---|---|---|
| Rules with negligible configuration interaction | $\Delta J = 0, \pm1$ $(0 \not\to 0)$ Parity change one electron jumping with $\Delta\ell = \pm1$ $\Delta n$ arbitrary | $\Delta J = 0, \pm1$ $(0 \not\to 0)$ No parity change no change in electron configuration i.e, for all electron $\Delta\ell = 0, \Delta n = 0$ | $\Delta J = 0, \pm1, \pm2$ $(0 \not\to 0)$ No parity change no change in electron configuration (or) one electron jump with $\Delta\ell = 0, \pm2$, $\Delta n$ arbitrary |
| LS coupling | $\Delta S = 0$ $\Delta L = 0, \pm1$ $(0 \not\to 0)$ | $\Delta S = 0$ $\Delta L = 0, \Delta J = \pm1$ | $\Delta S = 0$ $\Delta L = 0, \pm1, \pm2$ $(0 \not\to 0, 0 \not\to 1)$ |

**SternGerlach experiment :**

Force exerted on electron $F = \mu_z \frac{\partial B_z}{\partial z}$; $z = \frac{g_j m_j \frac{\partial B_z}{\partial z} \mu_B d^2}{8kT}$

$\cos\theta_{\bar{L},\bar{S}} = \frac{|\bar{J}|^2 - |\bar{L}|^2 - |\bar{S}|^2}{2|\bar{L}||\bar{S}|}$, $\mu_\ell = \mu_B\sqrt{\ell(\ell+1)}\,Am^2$

$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$, $g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$

$\mu_B = eh/4\pi m = 9.27 \times 10^{-24} Am^2$, $\mu_I = g_N e|\bar{I}|/2m_P$

$\partial B_z/\partial z$ = rate of change of field in z - direction
$\mu_B$ = Bohr magneton
$\mu_\ell, \mu_I$ = Orbital, nuclear magnetic dipole moment respectively
$g_J, g_N, g_F$ = Lande, nuclear, hyperfine g-factor resp.

## UV PHYSICS ACADEMY

AMP(8)

### Line width

Natural line width : $\delta\upsilon_n = \frac{1}{2\pi}\left[\frac{1}{\tau_i} + \frac{1}{\tau_k}\right]$

$\delta\omega_n = \frac{1}{\tau_i} + \frac{1}{\tau_k}$

Doppler line width : $\delta\upsilon_D = \frac{\upsilon_0}{C}\sqrt{\frac{2RT}{M}\ln 2}$

$\delta\upsilon_D = \frac{0.83\upsilon_0}{C}\sqrt{\frac{2RT}{M}}$

$\delta\upsilon_D \propto \upsilon_0$, $\delta\upsilon_D \propto \sqrt{T}$, $\delta\upsilon_D \propto \frac{1}{\sqrt{M}}$, $\delta\upsilon_D \propto v_{rms}$

$\Delta\upsilon_D = 7.16 \times 10^{-7}\, \upsilon_0 \sqrt{\frac{T}{M}}$ Hz

Collisional Broadening : $\Delta\upsilon_C = \frac{1}{\pi\ell_0}\sqrt{\frac{8RT}{\pi M}}$

$\Delta\upsilon_C = aP_B$, $P_B = N_B kT$

### Laser

Stimulated absorption : $\frac{dP_{ik}}{dt} = B_{ik}N_i\omega(\upsilon)$

Spontaneous emission: $\frac{dP_{ki}}{dt} = A_{ki}N_k$, $A_{ki} = \frac{1}{\tau_k}$

Stimulated emission : $\frac{dP_{ki}}{dt} = B_{ki}N_k\omega(\upsilon)$

Gain coefficient $\alpha = \frac{C^2}{8\pi\gamma_0^2\Delta\upsilon}(N_2 - N_1)A_{21}$

Gain must be positive

$N_k(t) = N_k(0)\, e^{-t/\tau_k}$, $\tau_k = 1/A_k$

Stability condition for optical resonator :

$0 \le \left(1 - \frac{d}{R_1}\right)\left(1 - \frac{d}{R_2}\right) \le 1$, $\upsilon = \frac{nC}{2d}$ n = 1,2,...

Relativechange in frequency $\Delta\upsilon_n/\upsilon_n = -\Delta\mu/\mu$

$\tau_i, \tau_k$ = mean life time of initial and final state ; $\omega, \upsilon, \upsilon_0$ = frequency; M = molecular weight; R = gas constant; T = temparature; $\ell_0$ = mean free path; $P_B$ = partial pressure; k = Boltzmann constant; dP/dt = probability rate; i, k = initial & final state; $A_{ki}$, $B_{ik}$ = Einstein coefficient for sponteneous emission, induced absorption, N = no. of atoms, $\gamma_0$ = pumping rate

## UV PHYSICS ACADEMY

AMP(5)

### Molecular Spectra

Total energy $E = E_e + E_r + E_v$ $(E_e > E_v > E_r)$

**Molecular Spectra**

| Rotational Spectrum | Vibrational Spectrum | Electronic Spectrum |
|---|---|---|
| 1. Transition between rotational energy states | Transition between vibrational state of molecule when electronic configuration remain same | Transition between two electronic energy states |
| 2. Levels are separated by order of $10^{-3}$eV | Levels are separated by order of $10^{-1}$eV | Levels are separated by orderof 1eV-10eV |
| 3. Lies in microwave (or) far IR region | Lies in near IR region | Lies in visible and UV region |

**Rotational spectra :**

$E_r = J(J+1)\hbar^2/(2I)$, $B = h/(8\pi^2 IC)\,cm^{-1}$, $\bar{\upsilon} = \frac{h}{(8\pi^2 IC)}J(J+1)\,cm^{-1}$; $J_{max} = \sqrt{\frac{KT}{2BhC}} - \frac{1}{2}$

Type of molecule

- Symmetric top: $I_a \neq I_b = I_c$
  - Prolate: $I_a < I_b = I_c$
  - Oblate: $I_b = I_a < I_c$
- Asymmetric: $I_a \neq I_b \neq I_c$
- Spherical: $I_a = I_b = I_c$

**For non-rigid rotor :**

$\epsilon(J) = B_e J(J+1) - D_e J^2(J+1)^2 + ...$

$D_e = \frac{4B_e^3}{\bar{\omega}^2}$, $\bar{\omega} = \frac{1}{2\pi C}\sqrt{\frac{k}{\mu}}$; $\Delta J = \pm1$

I = moment of inertia; T= temp. $\bar{\omega}$ = vibrational frequency; J = rotational quantum no.=0, 1,2...; K= Boltzmann const. B = rotational constant; $D_e$ = centrifugal distortion constant

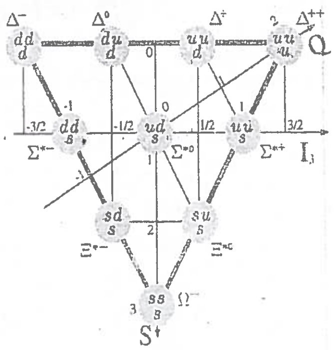
## UV PHYSICS ACADEMY

NP(14)

### PARTICLE PHYSICS

Spin 3/2 Baryons :

| Particles | Strangeness | Quark content |
|---|---|---|
| $\Delta^{++}$ | 0 | uuu |
| $\Delta^{+}$ | 0 | uud |
| $\Delta^{0}$ | 0 | udd |
| $\Delta^{-}$ | 0 | ddd |
| $\Sigma^{*+}$ | –1 | suu |
| $\Sigma^{*0}$ | –1 | uds |
| $\Sigma^{*-}$ | –1 | dds |
| $\Xi^{*0}$ | –2 | ssu |
| $\Xi^{*-}$ | –2 | dss |



**NP(1)** **NUCLEAR PHYSICS**

**Nuclear Properties**

$\vec{\mu}_\ell = g_\ell \mu_N \vec{\ell}, \ \vec{\mu}_s = g_s \mu_N \vec{s}$

Mass defect $\Delta M = M(A,Z) - A$

Binding energy B.E = $\Delta MC^2$, $\Delta M = ZM_H + NM_N - M(A,Z)$ ; Packing fraction $f = \frac{\Delta M}{A}$

Seperation energy : $S_p = \left|B.E(_Z X^A) - B.E(_{Z-1} Y^{A-1})\right|$; $S_n = \left|B.E(_Z X^A) - B.E(_Z Y^{A-1})\right|$

Radius $R = R_0 A^{1/3}$, Density $D = 10^{14}$ gm/cc

| Particle | Rest mass in kg | Rest mass in amu | Rest energy (eV) |
|---|---|---|---|
| Electron ($m_e$) | $9.109 \times 10^{-31}$ | $5.48580 \times 10^{-4}$ | $5.11003 \times 10^5$ |
| Proton ($M_p$) | $1.6726 \times 10^{-27}$ | 1.0072765 | $9.38280 \times 10^8$ |
| Neutron ($M_n$) | $1.67495 \times 10^{-27}$ | 1.0086650 | $9.39573 \times 10^8$ |

1u = 931.502 MeV, 1 fm = $10^{-15}$m, 1 barn = $10^{-28}$m², $R_0$ = 1.2 - 1.4 fm

**Magnetic moment** **Quadrupole moment**

$\langle\mu\rangle_{j=\ell+1/2} = \left[g_\ell\left(j - \frac{1}{2}\right) + \frac{g_s}{2}\right]\mu_N$

$\hat{Q} = 3Z^2 - r^2 = \frac{-3}{5}\frac{(2j-1)}{2(j+1)} r_0^2 A^{+2/3}$

$\langle\mu\rangle_{j=\ell-1/2} = \left[\frac{g_\ell j\left(j+\frac{3}{2}\right)}{(j+1)} - \frac{g_s}{2(j+1)}\right]\mu_N$

$\langle r^2 \rangle = \frac{3}{5} R_0^2 A^{2/3}$

A = mass number
M(A, Z) = atomic mass
C = velocity of light
$R_0$ = nuclear radius
$\mu_N$ = nuclear magneton = $5.0571 \times 10^{-27}$ J/T
j = nuclear spin

Proton: $g_\ell = 1, g_s = 5.59, \mu_p = 2.792\,\mu_N$

Neutron: $g_\ell = 0, g_s = -3.83, \mu_n = -1.913\,\mu_N$

Nuclear shapes: spherical (Q = 0) oblate (Q < 0) Prolate (Q > 0)

**NP(3)** **RADIOACTIVITY**

| Carbon dating | Uranium dating |
|---|---|
| $t = \frac{1}{\lambda} \ln \frac{N_0}{N}$ ($T_{1/2}$ of $C^{14}$ = 5760 yrs) | $t = \frac{1}{\lambda} \ln\left[1 + \frac{N_{pb}}{N_U}\right]$ ($T_{1/2}$ of $U^{238}$ = $4.5 \times 10^9$ yrs) |

$\alpha$ – decay : $^A_Z X \rightarrow ^{A-4}_{Z-2} Y + ^4_2 He$ ; $K.E_\alpha = Q\left[\frac{A-4}{A}\right]$; $K.E_y = Q\left[\frac{4}{A}\right]$

Gieger's law : $R = aE^{3/2}$ & $R \propto v^3$ ; for E in Mev & R in cm $\Rightarrow$ a = 0.315

Gieger – Nuttal law : $\log\lambda = A + B \log R$

**Tunnel theory of $\alpha$ – decay :**

Decay constant : $\lambda = N\,T$

Collision frequency : $N = \frac{v}{2R_0}$

$\beta$ decay : $\beta^{-1}: Q = (M_x - M_y)c^2$; $\beta^+ : Q = (M_x - M_y - 2m_e)c^2$

Electron capture : $Q = (M_x - M_y)c^2$

Q = energy released or obsorbed in the reaction.
N = no. of times per sec an $\alpha$–particle strike the barrie in a nucleus
T = transmition probability of the particle.
v = $\alpha$–particle velocity when it leaves the nucleus
$R_0$ = nucleur radius

**Transition rules :**

| Fermi Transition | Gamow – Teller Transition |
|---|---|
| $\Delta S = 0, \Delta J = 0, \Delta\pi$ = no change; electron and antineutrino spins are antiparallel. | $\Delta S = 1, \Delta J = 1, \Delta\pi$ = no change; electron and antineutrino spins are parallel. |

**UV PHYSICS ACADEMY**

**NP(5)** **DEUTERON PROBLEM**

| | | | | |
|---|---|---|---|---|
| Constituents | 1 p & 1 n | mass | 2.0147324u Ground state | $J^\pi = 1^+$ |
| Rms seperation | 4.2 fm | spin | $1\hbar$ Quadrupole moment | $2.73 \times 10^{-3}$ barn |
| Binding energy | 2.226 MeV | Magnetic moment | $0.85793\,\mu_N$ | |

Ground state Wave function of deuteron $\psi_d = \frac{u_0(r)}{r} Y^1_{011} + \frac{u_2(r)}{r} Y^1_{211}$ ; $(^3S_1(96\%) + ^3D_1(4\%))$

Nuclear forces : **Properties**

Charge symmetry : (n - n) = $(p - p)_{nuc}$

nature Charge independence : (n - n) = $(p - p)_{nuc}$ = (p - n)

Spin dependent
short range
non – central

attractive for aligned spins; repulsive for antiparallel spins

**NUCLEAR MODEL**

Semiemperical mass formula :

Binding Energy = $a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$

| | |
|---|---|
| Volume energy | $E_v = a_1 A$ |
| Surface energy | $E_s = -a_2 A^{2/3}$ |
| Coulomb energy | $E_c = -a_3 \frac{Z(Z-1)}{A^{1/3}}$ |
| Asymmetry energy | $E_a = -a_4 \frac{(A-2Z)^2}{A}$ |
| Pairing enery | $\delta = a_5 A^{-3/4}$ |

Stable isobar $Z' = \frac{A}{1.98 + 0.015 A^{2/3}} = \frac{A}{2 + \frac{a_3}{2a_4} A^{2/3}}$

Shell model : $V = \frac{1}{2}kr^2 + a\vec{\ell}\cdot\vec{s}$

$\langle \ell \cdot s \rangle = \frac{1}{2}[j(j+1) - \ell(\ell+1) - s(s+1)]\hbar^2$

$\Delta E = \langle \ell \cdot s \rangle_{j=\ell-\frac{1}{2}} - \langle \ell \cdot s \rangle_{j=\ell+\frac{1}{2}}$ ; $\Delta E = a(\ell + \frac{1}{2})\hbar^2$

$a_1 = 0.016919$ u, $a_2 = 0.0191144$ u,
$a_3 = 0.0007626$ u, $a_4 = 0.02544$ u,
$a_5 = 0.036$ u
$\delta$= 1 for even-even
0 for even-odd/odd-even

**UV PHYSICS ACADEMY**

**NP(7)**

Mean life time of excited state $\tau = \frac{\hbar}{\Gamma}$

$\sigma_{sc}(\theta) = |f(\theta)|^2$, $\sigma^{El} = \pi\lambda^2 \sum_{\ell=0}^{\infty}(2\ell+1)|1 - \eta_\ell|^2$

$\sigma^{Re} = \pi\lambda^2 \sum_{\ell=0}^{\infty}(2\ell+1)(1 - |\eta_\ell|^2)$ $\eta_\ell$ = 1 for elastic scattering

$\eta_\ell$ = 0 for shadow scattering

$\eta_\ell$ < 1 for reaction scattering

Shadow scattering : $\sigma^{El} = \sigma^{Re} = \pi\lambda^2 \sum_{\ell=0}^{\infty}(2\ell+1) = \pi R^2$ ; $\sigma_T = 2\pi R^2$

$\sigma_{sc}(\theta)$ = elastic scatter cross-section p unit solid angle
$f(\theta)$ = scattering amplitude
El = elastic,
Re = reaction
R = radius of nucleus
$a_t$ = triplet scattering ler
$a_s$ = singlet scattering le

**Optical theorem :** $\sigma^{total} = \frac{4\pi}{K} \text{Im}\, f(0)$

Ortho Hydrogen : Two protons has spin in same direction

Para Hydrogen : Two protons has spin in opposite direction

Sacttering length : $a = a_t\pi_t + a_s\pi_s$

$\sigma_n \cdot \sigma_p = \begin{cases} -3 \text{ for single} \\ 1 \text{ for triplet} \end{cases}$

Projection operators : $\pi_t = \frac{1}{4}[3 + \sigma_n \cdot \sigma_p]$; $\pi_s = \frac{1}{4}[1 - \sigma_n \cdot \sigma_p]$

$\sigma_{ortho} = \frac{1}{4}[3a_t + a_s]^2 + 2(a_t - a_s)^2$, $\sigma_{para} = \frac{1}{4}[3a_t + a_s]^2$; $\sigma_{ortho} > \sigma_{para}$, $\frac{\sigma_{ortho}}{\sigma_{para}} = 1 + \frac{8(a_t - a}{(3a_t + a}$

**UV PHYSICS ACADEMY**

**NP(9)** **PARTICLE PHYSICS**

Strong interaction : $\Delta S = 0, \Delta I = \Delta I_3 = 0$

Weak decay :
- Leptonic decay $\Delta S = \pm 1$ $\Delta Q = \Delta S$
- Non leptonic decay $\Delta S = \pm 1, |\Delta S| = |\Delta I| = |\Delta I_3|$

Reaction threshold kinetic energy :

$K_{th} = -Q \frac{\text{total mass of all particles involved in reaction}}{2 \times \text{mass of target particle}}$

Type of interaction :

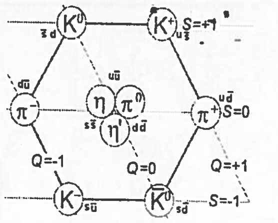
| Interaction | Characteristic constant | Strength | Range | Typical cross-section | Life time |
|---|---|---|---|---|---|
| Strong | $\frac{g_h^2}{\hbar c}$ | 1 ~ 10 | $10^{-13}$ cm | $10^{-26}$ cm² | $10^{-23}$ sec |
| E.M. | $\frac{e^2}{\hbar c} = \alpha$ | 1/137 | $\infty$ | $10^{-29}$ cm² | $10^{-16}$ sec |
| Weak | $\frac{g_w^2}{\hbar c} = \frac{Gm_p^2 c}{\hbar^3}$ | $10^{-5}$ | $10^{-16}$ cm | $10^{-38}$ cm² | $10^{-10}$ sec |
| Gravitational | $\frac{Gm_p^2}{\hbar c}$ | $10^{-39}$ | $\infty$ | - | - |

**UV PHYSICS ACADEMY**

**NP(11)** **PARTICLE PHYSICS**

**Elementary particles and forces :**

| Particles | Weak | E.M. | Strong |
|---|---|---|---|
| Quarks | Y | Y | Y |
| Charged leptons | Y | Y | N |
| Chargeless leptons | Y | N | N |

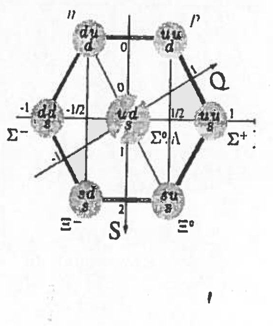



**Quarks :**

| Symbol | Q/e | Spin | Baryon No. | Strangeness |
|---|---|---|---|---|
| u | 2/3 | 1/2 | 1/3 | 0 |
| d | –1/3 | 1/2 | 1/3 | 0 |
| c | 2/3 | 1/2 | 1/3 | 0- |
| s | –1/3 | 1/2 | 1/3 | –1 |
| t | 2/3 | 1/2 | 1/3 | 0 |
| b | –1/3 | 1/2 | 1/3 | 0 |

**Quarks Structure :**

| Quarks | Q | I | $I_3$ | S | B | Y |
|---|---|---|---|---|---|---|
| u | 2/3 e | 1/2 | 1/2 | 0 | 1/3 | 1/3 |
| d | –1/3 e | 1/2 | –1/2 | 0 | 1/3 | 1/3 |
| s | –1/3 e | 0 | 0 | –1 | 1/3 | –2/3 |



Non mirror decays are GT – dominant (≥ 98% GT);

$\beta$ – decay mechanism and parity (transition from $i \to f$)

$v_{fi} = \int \psi_f^* \, v \, \psi_i \, d\tau$ (integrand must have even parity)

| V | | parity |
|---|---|---|
| Scalar | scalar spatial part is constant | same parity |
| Scalar | spatial part is polar vector $\vec{r}$ | different parity |
| Axilar vector | Eg: $\vec{\ell} = \vec{r} \times \vec{p}$ | same parity |
| Pseudo Scalar | (scalar product of polar and axial vector) | different parity |
| Vector | | same parity |

$\Gamma$ = width of the level
$\phi$ = flux of the beam
Ax = vol. of the sample
$\sigma$ = cross-section

Multipole radiation:
Electric : Parity $\pi = (-1)^L$ ;
Magnetic : Parity $\pi = (-1)^{L+1}$
L = 1(dipole),
L = 2(quadrapole),
L = 3(octopole),
L = 4 (hexa decapole)

$\gamma$ decay : (No change in Z, N or A);

Intensity of $\gamma$ – rays : $I = I_0 e^{-\mu x}$, $\mu = \frac{\ln 2}{d_{1/2}}$ ;

Transition probability $T = \lambda = \frac{1}{\tau} = \frac{\Gamma}{\hbar}$

Mean free path $\lambda = \frac{1}{n\sigma}$ ; Reaction rate $\frac{\Delta N}{\Delta t} = \phi n^l \sigma$, $n^l = nAx$

number of $\alpha$ - particles :
$X_{Z_1}^{A_1} \to Y_{Z_2}^{A_2}$
no. of $\alpha$ particles = $\frac{A_1 - A_2}{4}$
no. of $\beta$ particles =
$Z_2 - [Z_1 - 2(\text{no. of } \alpha \text{ particles})]$

Half life $T_{1/2} = \frac{\ln 2}{\lambda}$, Mean life $\tau = \frac{1}{\lambda} = 1.44 T_{1/2}$

$\frac{N}{N_0} = \frac{1}{2^x}$, $x = \frac{T}{T_{1/2}}$, Acitivity $A = A_0 e^{-\lambda t}$ ;

Successive disintegrations: $A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$

$N_2 = \frac{\lambda_1 N_0}{\lambda_1 - \lambda_2}[\exp(-\lambda_2 t) - \exp(-\lambda_1 t)]$, $t_m = \frac{1}{\lambda_1 - \lambda_2} \ln\left[\frac{\lambda_1}{\lambda_2}\right]$

$\lambda_2$ = decay constant of B
dps = decay per second
G = giga
$\frac{N_0}{N}$ = ratio of $C^{14}$ at time of death to $C^{14}$ at time of finding age.

Equilibrium
- Secular : $T_{1/2}$ of parent >>> $T_{1/2}$ of daughter : $\lambda_1 N_1 = \lambda_2 N_2$
- Transient : $T_{1/2}$ of parent > $T_{1/2}$ of daughter : $\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1}$

Branching of radioactivity : $\frac{1}{\tau} = \frac{1}{\tau_\alpha} + \frac{1}{\tau_\beta}$

I curie (Ci) = $3.7 \times 10^{10}$ dps, 1 Becquerel = 1 dps, 1 Rutherford (Ru) = $10^6$ dps

| Series | Name | Parent | End product | Half Life |
|---|---|---|---|---|
| 4n | Thorium | $_{90}Th^{232}$ | $_{82}Pb^{208}$ | 14 Gyrs |
| 4n + 1 | Neptunium | $_{93}Np^{237}$ | $_{83}Bi^{209}$ | 2.1 Gyrs |
| 4n + 2 | Uranium | $_{92}U^{238}$ | $_{82}Pb^{206}$ | 4.47 Gyrs |
| 4n + 3 | Actinium | $_{92}U^{235}$ | $_{82}Pb^{207}$ | 0.7 Gyr |

## UV PHYSICS ACADEMY
### PARTICLE PHYSICS

P(8)

Range of interaction : $\Delta x = \frac{197 \text{MeV fm}}{E}$

I = isospin
M = multiplicity
Q = charge of particle
$\overline{Q}$ = average charge of the group
S = strangness number

Elementary particles
- Leptons: $\begin{pmatrix} e^- \\ \upsilon_e \end{pmatrix}$ $\begin{pmatrix} \tau^- \\ \upsilon_\tau \end{pmatrix}$ $\begin{pmatrix} \mu^- \\ \upsilon_\mu \end{pmatrix}$
- Quarks
  - Measons (2Q)
    - $\pi$: $\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$
    - K: $\begin{pmatrix} k^+ \\ k^0 \end{pmatrix}$
    - $\eta$: $(\eta^0)$
  - Baryons (3Q)
    - Nucleons: $\begin{pmatrix} p \\ n \end{pmatrix}$
    - Hyperons: $\Lambda^0$, $\begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}$, $\begin{pmatrix} \Xi^- \\ \Xi^0 \end{pmatrix}$, $\Omega^-$

Lepton no. : L = 1 for leptons; –1 for anti leptons; 0 all others

Baryon no. : B = 1 for baryons; –1 for anti baryons; 0 all others

Isospin : M = 2I + 1; third component of isospin : $I_z = Q - \overline{Q}$

Gellman & Nishijiman relation : $Q = I_3 + \frac{B+S}{2}$ ; Hypercharge : Y = S + B

## UV PHYSICS ACADEMY

NP(6)

$\Delta\epsilon_{\ell,s} = 10(2\ell+1)A^{-2/3}$ MeV ; Magic numbers : 2, 8, 20, 28, 50, 82, 126 and 184

Nuclear spin = $J^\pi$, $\pi$ = parity = $(-1)^\ell$

Nordhiem number : $N = j_p - \ell_p + j_n - \ell_n$ ; (It is used to determined spin of odd-odd nuclei)

Nordhiem's rule
- If $j_n = \ell_n + \frac{1}{2}$ & $j_p = \ell_p - \frac{1}{2} \to N = 0$ or vice versa
- If $j_n = \ell_n + \frac{1}{2}$ & $j_p = \ell_p + \frac{1}{2}$ (or) $j_n = \ell_n - \frac{1}{2}$ & $j_p = \ell_p - \frac{1}{2}$ ; $N = \pm 1$

Strong rule : N = 0, $I = |j_n - j_p|$; Weak rule : $N = \pm 1$ , $J = |j_n - j_p|$ or $|j_n + j_p|$

Finding Spin
- N & Z both even J = 0
- For odd A, J is determined by j value of last odd nucleon
- For odd Z & N, J is found by nordhiem rule

Coulomb energy = $\frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{ze^2}{r_0 A^{1/3}}$
J = nuclear spin

Reactions
- Exoergic (Q > 0)
- Elastic scattering (Q = 0)
- Endoergic (Q < 0)

Nuclear reaction :
$X + x \to Y + y$
$Q = M_X + M_x - M_Y - M_y$

$Q = E_y\left[1 + \frac{M_y}{M_Y}\right] - E_x\left[1 - \frac{M_x}{M_Y}\right] - \frac{2}{M_Y}\sqrt{M_x M_y E_x E_y}\cos\theta$

Threshold kinetic energy : $E_{th} = |Q|\left[1 + \frac{m}{M}\right]$

## UV PHYSICS ACADEMY
### PARTICLE PHYSICS

IP(12)

| Anti Quarks | Q | I | $I_3$ | S | B | Y |
|---|---|---|---|---|---|---|
| $\bar{u}$ | –2/3 | 1/2 | –1/2 | 0 | –1/3 | –1/3 |
| $\bar{d}$ | 1/3 | 1/2 | 1/2 | 0 | –1/3 | –1/3 |
| $\bar{s}$ | 1/3 | 0 | 0 | 1 | –1/3 | 2/3 |

| Particle | Parity |
|---|---|
| Nucleon | even |
| Mesons | odd |
| Hyperons | even |

Mesons (spin = 0):

| Particle | Mass (MeV/c$^2$) | Strangeness | Mean Life time (sec) | Typical decay | Quark content |
|---|---|---|---|---|---|
| $\pi^0$ | 135.0 | 0 | $8.4\times10^{-17}$ | $\gamma+\gamma$ | $u\bar{u}, d\bar{d}$ |
| $\pi^+$ | 139.6 | 0 | $2.6\times10^{-8}$ | $\mu^+ + \upsilon_\mu$ | $u\bar{d}$ |
| $\pi^-$ | 139.6 | 0 | $2.6\times10^{-8}$ | $\mu^- + \bar{\upsilon}_\mu$ | $\bar{u}d$ |
| $K^+$ | 493.7 | 1 | $1.24\times10^{-8}$ | $\mu^+ + \upsilon_\mu$ | $u\bar{s}$ |
| $K^-$ | 493.7 | –1 | $1.24\times10^{-8}$ | $\mu^- + \bar{\upsilon}_\mu$ | $\bar{u}s$ |
| $\eta^0$ | 547.3 | 0 | $\cong 10^{-18}$ | $\gamma+\gamma$ | $u\bar{u}, d\bar{d}\ s\bar{s}$ |

## UV PHYSICS ACADEMY
### PARTICLE PHYSICS

NP(10)

Mediating particle :

| Interaction | Exchanging particle | mass | Charge | Spin |
|---|---|---|---|---|
| Strong | Gluons | 0 | 0 | 1 |
| E.M. | Photons | 0 | 0 | 1 |
| Weak | $W^\pm$ , | 81 GeV/c$^2$ | ±e | 1 |
| | $Z^0$ | 93 GeV/c2 | 0 | 1 |
| Gravitational | Gravitons | 0 | 0 | 2 |

Leptons :

| Particle Name | Symbol | Anti particle | Mass (MeV/c$^2$) | Life time (sec) | Principal decay mode |
|---|---|---|---|---|---|
| Electron | $e^-$ | $e^+$ | 0.511 | stable | |
| Electron neutrino | $\upsilon_e$ | $\bar{\upsilon}_e$ | $< 3\times10^{-6}$ | stable | |
| Muon | $\mu^-$ | $\mu^+$ | 105.7 | $2.2\times10^{-6}$ | $(e^-, \bar{\upsilon}_e, \upsilon_\mu)$ |
| Muon neutrino | $\upsilon_\mu$ | $\bar{\upsilon}_\mu$ | < 0.19 | stable | |
| Tau | $\tau^-$ | $\tau^+$ | 1777 | $2.9\times10^{-13}$ | $(\mu^- \bar{\upsilon}_\mu \upsilon_\tau / e^- \bar{\upsilon}_e \upsilon_\tau)$ |
| Tau neutrino | $\upsilon_\tau$ | $\bar{\upsilon}_\tau$ | < 18.2 | stable | |